An Adaptive Grid-Based All Hexahedral Meshing Algorithm Based on 2-Refinement

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Abstract. Most adaptive mesh generation algorithms employ a 3-refinement method. This method, although easy to employ, provides a mesh that is often too coarse in some areas and over refined in other areas. Because this method generates 27 new hexes in place of a single hex, there is little control on mesh density. This paper presents an adaptive all-hexahedral grid-based meshing algorithm that employs a 2-refinement method. 2-refinement is based on dividing the hex to be refined into eight new hexes. This method allows a greater control on mesh density when compared to a 3-refinement procedure.

1 Introduction

The ability to adapt (i.e. coarsen and refine) all hexahedral meshes is an area of current research [1]-[3]. A promising solution to the adaptive hex-based meshing algorithm problem includes grid-based methods [1], [4]-[6]. Most octree methods are based on 3-refinement. 3-refinement involves splitting a selected octant within an octree three times along one edge. For three dimensions this involves uniformly splitting a single hex into 27 new hexes as shown in figure 1. This 1-to-27 split often results in meshes containing highly coarse and highly refined sections. 2-refinement involves splitting the edge of an octant twice and, in three dimensions, results in a 1-to-8 split, also shown in figure 1. This refinement provides greater control on mesh density and allows a much smoother transition from areas of higher to lower mesh densities. However, 3-refinement is much easier to implement because all refinement can be maintained within a single element sheet (see reference [7] for a description of hexahedral element duals). 2-refinement requires a set of parallel sheets and then additional modifications beyond these two sheets. A comparison of these issues, as related to 2- and 3- refinement is shown in figure 2.

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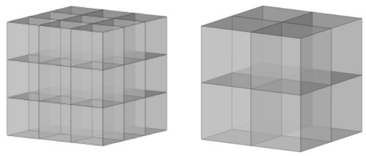

Fig. 1. Hexahedral 3-refinement and 2-refinement

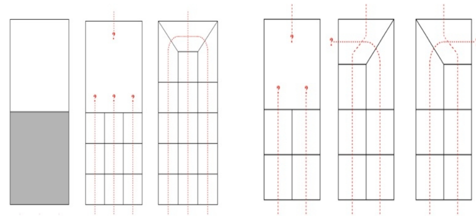

Fig. 2. Two dimensional comparison of 3- and 2-refinement

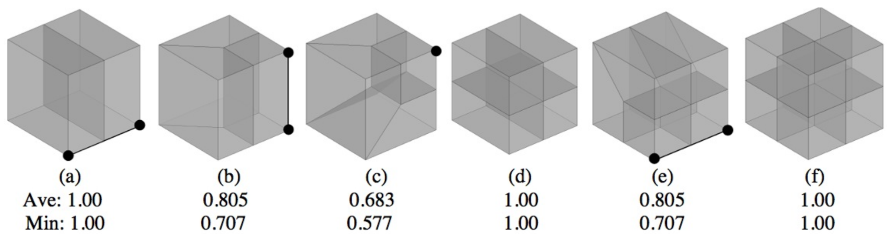

Fig. 3. 2-Refinement Templates and contained element qualities

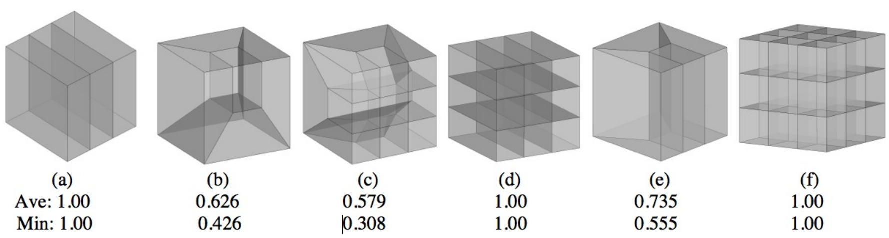

Fig. 4. 3- Refinement Templates and contained element qualities

Grid-based algorithms often employ templates for the insertion of conformal elements into a transition layer between the uniform refinement zone and the original hexes [4], [8]. Figure 3 shows the 2-refinement templates with their associated average and minimum scaled jacobian values. The scaled jacobian is a quality measure often used to evaluate element quality [9]. Common 3-refinement templates are shown in figure 4 along with their minimum and average scaled jacobian values. As shown, element quality with 2-refinement templates is substantially higher than 3-refinement templates with the minimum scaled jacobian value of 0.5774 compared to 0.3077.

2 Algorithm Development

The algorithms developed to implement 2-refinment for structured hexahedral meshes are completely described in reference [10]. We choose to develop the algorithms by first considering refinement only in one coordinate direction, followed by consideration of refinement in two coordinate directions and completed by a full three directional refinement. These algorithms feature three important enhancements to the existing state of the art that are 1) staggered transition layers, 2) a new template for concave regions in two directional refinement and 3) a new template for concave regions in three directional refinement. A comparison of effective quality using staggered transition layers is shown in figures 5 and 6 and the concave templates for two and three directional refinement are shown in figures 7 and 8.

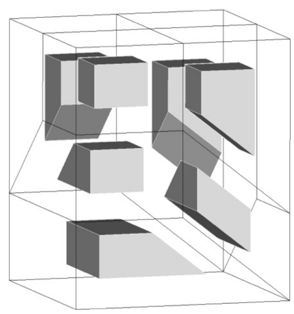

Fig. 5. Without Staggering (Ave: 0.704, Min: 0.462)

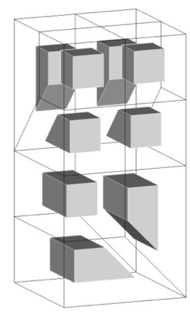

Fig. 6. With Staggering (Ave: 0.805, Min: 0.707)

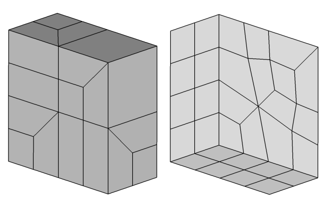

Fig. 7. Concave 2-directional template showing front and back views

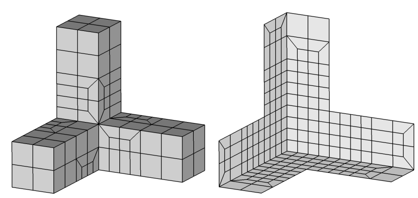

Fig. 8. Concave 3-directional template showing front and back views

3 Examples

Examples demonstrating the 2-refinement of structured hexahedral meshes are given below in figure 9. Shown is a geological region refined first by the two directional algorithm and second by a three directional algorithm. Note that the first example has a concave region. Tables 1 and 2 provide the element counts and quality of the meshes created. The iteration column identifies the number of transitions included.

Table 1. Results of Two Directional 2- Refinement Example

Iter	Elems	Ave.	Std. Dev.	Min.	Max.
0	3,240	0.973	0.048	0.515	1.0
1	11,208	0.911	0.121	0.404	0.999
2	50,520	0.898	0.136	0.346	0.998
3	282,360	0.910	0.114	0.269	0.998

Table 2. Results of Three Directional 2- Refinement Example

Iter	Elems	Ave.	Std. Dev.	Min.	Max.
0	3,072	0.960	0.055	0.463	1.0
1	7,900	0.904	0.124	0.420	1.0
2	14,456	0.864	0.125	0.373	1.0
3	21,156	0.859	0.131	0.305	1.0

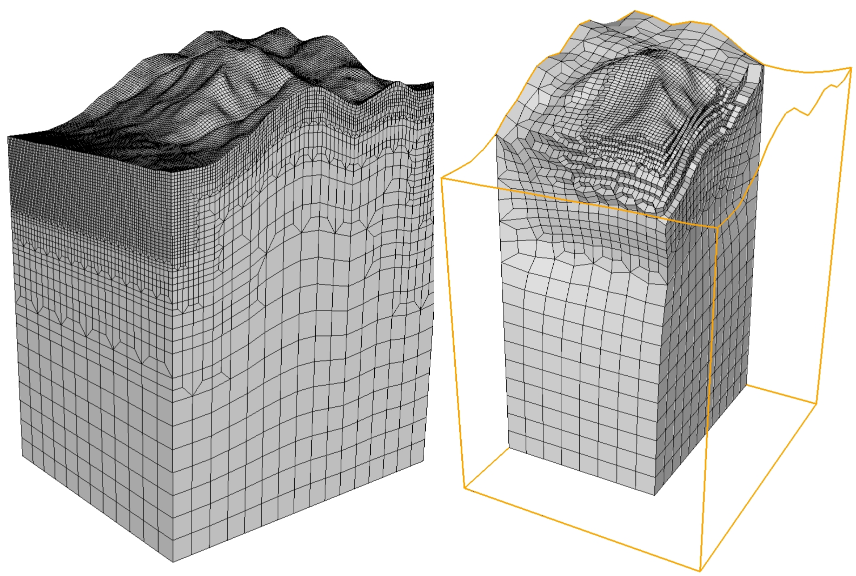

Fig. 9. Geotechnical example showing Two- and Three- Directional 2-Refinement

4 Conclusions

This adaptive all-hexahedral meshing algorithm provides a mesh that is efficient for analysis by providing a high element density in specific locations and a reduced mesh density in other areas. In addition, this tool can be effectively used for inside-out hexahedral grid based schemes, using Cartesian structured grids for the base mesh, which have shown great promise in accommodating automatic all-hexahedral algorithms. This adaptive all-hexahedral grid-based meshing algorithm employs a 2-refinement insertion method. This allows greater control on mesh density when compared to 3-refinement methods. This algorithm uses a two layer transition zone to increase element quality and keeps transitions from lower to higher mesh densities smooth. Templates were introduced to allow both convex and concave refinement.

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