Hybrid Mesh Generation for Reservoir Flow Simulation in CPG Grids

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Abstract. In this paper, we introduce a new method to generate a hybrid mesh from a CPG (Corner Point Geometry) reservoir grid and a radial mesh around a well. The method is an extension of the approach proposed in [1] to the case of CPG grids with high level of deformation. This ensures a fully functional mesh generation for realistic cases. The main idea is first to construct a mapping between the real space containing the CPG grid along with the radial mesh of the well and a virtual space where the CPG reservoir grid becomes a Cartesian grid. Then, because this mapping damages the circular property of the radial mesh, an appropriate radial mesh is built in the virtual space and the initial mapping is modified by taking into account the new radial mesh in the virtual space. To this end, an optimization technique using mesh refinement procedures is applied. The mapping combined with the mentioned deformation allows us to generate an unstructured polyhedral transition mesh (between the reservoir grid and the radial mesh) in the virtual space using the algorithm proposed in [1]. Finally, coming back to the real space, the obtained hybrid mesh may require a post processing step to recover the requested finite volume properties.

Introduction

The new technological improvements in 3D seismic imagery and drilling/production enable today to obtain a realistic and faithful image of the internal architecture of the reservoir and to drill deviated and complex 3D wells with several levels of ramification. Well trajectories can be well adapted to the geometry of the reservoir in order to optimize its production. In this new technological context, the mesh generation becomes a crucial step in the reservoir flow simulation of new generation. Meshes allow us to represent the geometry of the geological structures with discrete elements on which the simulation is processed. A better comprehension of the physical phenomena requires us to simulate 3D multiphase flows in increasingly complex geological structures, in the vicinity of several types of singularities such

as complex wells. All these complexities must initially be taken into account within the mesh construction and the mesh must faithfully represent all this heterogeneous information.

Whereas the classical meshes are totally structured or unstructured, in [5], a hybrid mesh model was proposed in 2D to capture the radial characteristics of the flow around the wells. It combines the advantages of the structured and unstructured approaches, while limiting their disadvantages. In [3], the generation methodology was extended to 3D case where the reservoir grid is Cartesian. The hybrid mesh is composed of a structured hexahedral mesh describing the reservoir field, structured radial meshes adapted locally to flow directions around each well and unstructured polyhedral meshes (based on power diagrams) connecting the two structured meshes.

In this paper, the generation of the hybrid mesh is extended to the 3D highly deformed cases. In Section 1, we present the numerical constraints imposed by finite volume schemes that will govern the mesh construction. In Section 2, we quickly recall the methodology used to generate a hybrid mesh in 3D and introduce the problems induced by the cartesian methodology in the real reservoirs grids. A solution to generate a hybrid mesh in such a grid is given in section 3.

1 Problem Statement

We present here a quick overview of the problem. In order to achieve numerical simulations of a phenomenon, we need first to establish a mathematical model of the phenomenon. This model is then discretized in order to define a numerical scheme. Finally, this numerical scheme is applied to a grid which is the discretization of the space where the phenomenon occurs.

1.1 Mathematical Model of Flow Simulation

The reservoir simulation is the whole set of operation allowing the modelisation behaviour of a petroleum reservoir. The aim of this simulation is to drive the reservoir exploitation and to argue the different technical choices to make. In a petroleum reservoir, flows are the consequences of variations in space and time of pressure and saturation in the water, oil and gas phases. These variations are induced by the production or the injection of one of this fluids into the different wells. The mathematical modelisation of the flow allows to take into account simultaneously the whole set of physical relationships describing the interactions inside the medium. Let us consider the case of a compressible, diphasic flow without capillary pressure (the different phases have the same pressure) at a constant temperature and without chemical reaction. The global equation can be written as follows:

$$\rho \frac{\mathrm{d}P}{\mathrm{d}t} + div(\rho \frac{\mathsf{K}}{\mu} \, (-\mathbf{grad}\ \mathbf{P} + \rho\ \mathbf{g})) = 0\,. \tag{1}$$

where P is the pressure, ρ the density of the fluid, $\mathbf{g}$ the gravity, K the permeability tensor and μ the viscosity of the fluid.

1.2 Numerical Schemes

Most common numerical schemes used in reservoir flow simulation are volume finite schemes. They use the mass conservation law for two adjacent cells. Each discretization point is associated with a control volume. This control volume is defined by a set of bounding faces.

Considering that the pressure in a cell is constant, for a face $\partial C_1 \cap \partial C_2$ separating a cell C_1 to a cell C_2, if the face normal $\mathbf{n} \simeq \frac{\mathbf{o_1 o_2}}{o_1 o_2}$, then we can approximate the flow between both cells through the face with area A_i by

$$-\int_{\partial C_1 \cap \partial C_2} \mathsf{K}\ \mathbf{grad\ p} \simeq A_i\ \mathsf{K}\ \frac{p_{C_2} - p_{C_1}}{o_1 o_2}\ \mathbf{n}\,. \tag{2}$$

This way to approximate the flow requires meshes to respect some conditions:

1. **Mesh conformity**, ensuring a face to face connection.
2. **Mesh orthogonality**, ensuring that the normal vector of the face is colinear to the line joining the centers of the adjacent cells.
3. **Face planarity**, so that the flow approximation is more precise.
4. **Auto centering**, ensuring that the cell center lies inside the cell.

1.3 Meshes Overview

In order to represent the reservoir using discrete elements, the hybrid mesh is composed of three kinds of elementary meshes:

- A structured CPG grid, respecting the geological features, is used to describe the reservoir field.
- To gain accuracy at the drainage areas, a structured radial circular mesh adapts locally to the radial nature of the flows around the wells.
- Finally, these structured meshes are connected together by the use of unstructured polyhedral mesh with respect to conformity and finite volume properties.

While the structured grid generation is a well known process, the construction of the unstructured transition mesh in 3D constitutes a major issue. The structured CPG mesh of the reservoir grid is constructed through the use of transfinite interpolations, projections onto the geological interfaces (horizons and faults) and a relaxation procedure [2]. The structured radial mesh is computed given the well's trajectory, the drainage area radius and the progression of cells' size. The separate construction of these grids leads to incompatibilities due to a lack of common structure and a transition mesh is needed to carry out a correct connection.

Then, to generate such a transition mesh, a method using *power diagrams* was introduced in [5]. As a generalisation of Voronoï diagrams, power diagrams provide polyhedral, convex, and orthogonal cells. In addition, these allow here to reach the mesh conformity which would not be possible using Voronoï diagrams. In [3], the generation methodology was extended to 3D case where the reservoir grid is Cartesian.

2 Hybrid Meshes and Non Cartesian Grids

2.1 General Methodology and Its Restriction

The general methodology used to build an hybrid mesh from a reservoir grid and a radial mesh has been detailled in [1]. This methodology consists of the following steps:

1. the definition of the volume (called *cavity*) inside which will be built the polyhedral mesh and the boundaries of this volume,
2. the definition of the centers (called *sites*) of the polyhedral cells,
3. the weighted Delaunay triangulation of the sites previously defined and the polyhedral mesh computation.
4. the adjustment of the polyhedral mesh using topological considerations.

This methodology especially details how to link a radial mesh to a cartesian grid modelling the reservoir by the use of a polyhedral transition mesh. It is important to note that using this methodology must be restricted to the Cartesian reservoir grids. Unfortunately, the Cartesian modelling of a reservoir is not enough to take into account all its geological complexities. On the other hand, the Corner Point Geometry grids are able to handle non geological caracteristics like faults, sedimentation and erosion much more realistically. The problem occurring with such grids is that they do not respect some essential conditions for the hybrid mesh generation, in particular the planarity of the cavity faces, the cocyclicity of the cavity faces and the Delaunay admissibility of the cavity edges.

The methodology used to build the radial mesh is such that it ensures this properties are respected. That is why, cavity faces resulting from the radial mesh are always valid. In contrary, CPG grids are composed of quadrilateral faces whose vertices are not planar and thus non co-circular. Furthermore, this kind of grid does not ensure the Delaunay admissibility of the cavity edges, inherent properties of cartesian grids. Consequently, the methodology developped for the cartesian reservoir grid is not applicable to the CPG reservoirs.

2.2 A Solution Using Grid Deformation

At first, (because we will often talk about it), it is important to explain what we call *grid deformation*. The undeformed grid is the cartesian grid. In a

CPG grid, the cell faces are aligned with the geological structures of the real reservoir. By doing such thing, the cells of the reservoir are modified against the cartesian position, so that they are deformed. Quantification of such a deformation is not what we care about, whereas we need to ensure that all edges of the mesh are Delaunay admissible.

A grid deformation approach was proposed in [1]. This previous work consisted in deforming the CPG grid of the reservoir in order to make it cartesian. The aim was to restore the co-cyclic properties of the external faces of the *core*. This approach is based on two different spaces : a space called *real* ($\mathcal{E}_{real}$) in which the reservoir mesh is CPG and a space called *reference* ($\mathcal{E}_{reference}$), image of $\mathcal{E}_{real}$ where this one has a cartesian geometry. A bijective function ϕ ensures the definition of the image of one point from $\mathcal{E}_{real}$ to $\mathcal{E}_{reference}$.

When the reservoir grids are highly deformed, the radial mesh image computed using ϕ is too deformed and the essential conditions are not fulfilled anymore. The polyhedral mesh generation can not be achieved directly using the cartesian methodology, or the resulting mesh will not correspond to the cavity boundaries topology. The aim of the work we are talking in this paper is to develop a methodology which restore the necessary geometrical properties of the radial mesh surface. The recovery is achieved by an extension of the function ϕ computation.

3 Hybrid Meshes for Real Grids with High Deformation

As we explained in the previous section, the solution based on the grid deformation improve the external boundary of the cavity. Unfortunately, this solution impacts the radial grid which suffers the reservoir grid deformation. We present here an extension of this method which ensures that both radial grid and reservoir grid remain optimal for power diagram generation. To do so, we first modify the way defining the interpolation space, and we develop a set of tools allowing us to compute the intended mapping.

3.1 Mapping the Cavity Space

In the following, we define the *core* as the whole set of simplices covering the cavity intended to be filled by the polyhedral mesh.

The global definition of the function ϕ is the sum of the elementary functions ϕ_i defined inside each element of the *core*. An elementary function ϕ_i is defined as follows : Let P be a point contained by a tetrahedron T ($volume(T) \neq 0$). A, B, C and D are the tetrahedron vertices. P can be written as a linear combination of the four vertices:

$$P = \alpha A + \beta B + \gamma C + \delta D \tag{3}$$

where α, β, γ et δ are the barycentric coordinates of point P associated with A, B, C and D. The global function ϕ is the combination of all the elementary

functions. Ensuring that the different tetrahedra do not overlap, the global function is also bijective on the whole *core*. Knowing the barycentric coordinates of a point in a tetrahedron T, the computation of its new position is obvious. Thus, we have a simple piecewise linear interpolation function that can be defined on any arbitrary volume.

The aim of our methodology is to find a bijective mapping between both $\mathcal{E}_{real}$ and $\mathcal{E}_{reference}$ spaces. In the $\mathcal{E}_{reference}$ space, both surfaces on which will fit the transition mesh have to fulfill the geometrical requirements. The whole methodology is illustrated in Figure 1.

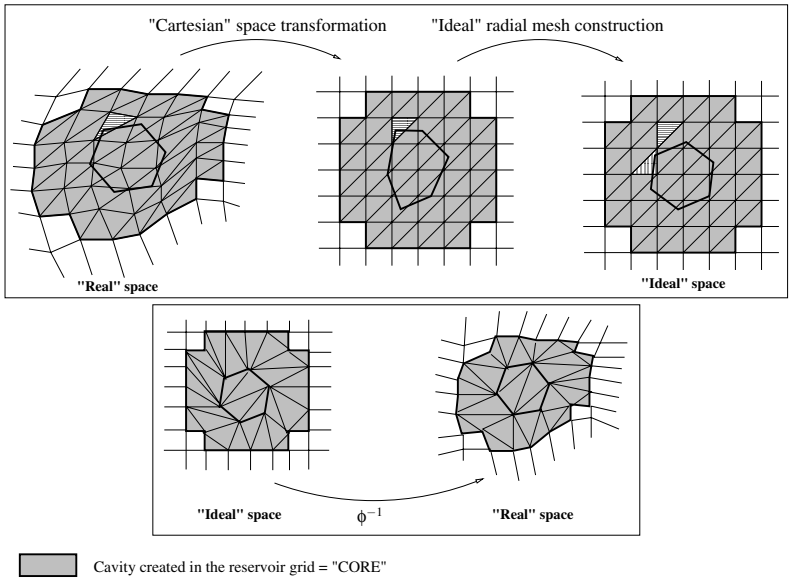

Fig. 1. Overview of the full methodology.

The first step was done by mapping the real reservoir grid to a non uniform reservoir grid which provide the $\mathcal{E}_{reference}$ space. This mapping is a temporary snapshot of the function ϕ, but properties of radial mesh are not preserved. The problem is that using this mapping, the obtained radial mesh is not suitable to apply the polyhedral mesh generation algorithm. Thus, we start defining a way to obtain an ideal radial mesh in the $\mathcal{E}_{reference}$ space. In reservoir modelling, physical wells are simply modelled by a curve following the trajectory of the well. In order to obtain a surface with the good properties, a simple idea is to build another radial mesh from the update trajectory ($= \phi$(initial trajectory)) of the well in the reference space. This process allows us to obtain a new external surface. The reconstruction method is the standard methodology used to build a radial mesh. This methodology is a simple *sweep* of a circular section along the well trajectory. We now call

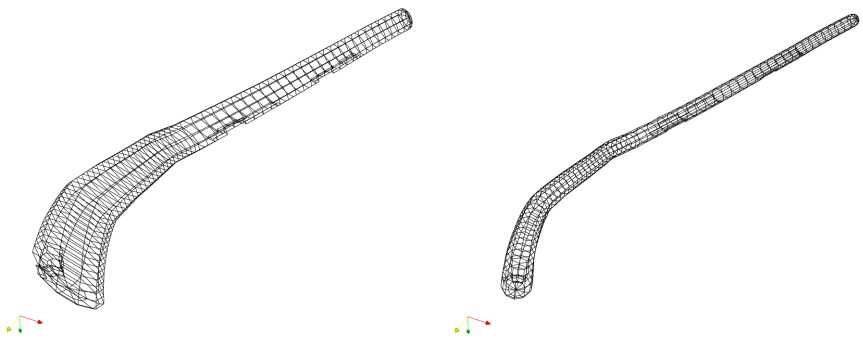

Fig. 2. Deformed radial mesh surface and its image using reconstruction.

$\mathcal{S}_{radial}$ the ideal surface obtained after radial mesh reconstruction. Both deformed and ideal surfaces (see Figure 2) have the same topology, i.e. the same connectivity between vertices.

At this point, we have the two different geometries which correspond to the surfaces in the real and the ideal spaces. The main objective is then to map the cavity volume using a function giving us the position of a point in one of the two spaces, knowing the position in the other space. It is obvious that vertices of $\mathcal{S}_{radial}$ do not conform to the initial mapping (see Figure 1, a specified vertex of the deformed $\mathcal{S}_{radial}$ is inside the horizontally hatched triangle whereas the same vertex of the ideal $\mathcal{S}_{radial}$ is inside the vertically hatched triangle). To correct the mapping, the simplest idea is to replace the tetrahedra obtained from the reservoir cells by an other set of tetrahedra obtained from the Delaunay triangulation of the vertices of the cavity boundaries. This ensures that the space is mapped with a set of tetrahedra without self intersection neither void and provides us with a piecewise representation of the cavity. In order to ensure the consistency of the mapping function, the cavity boundary must be preserved in the triangulation. This can be realized by generating the Delaunay triangulation in the ideal space. In the case where the deformation of the reservoir grid is small, the Delaunay triangulation remains valid in the real space. We thus have the space mapped with a bijective piecewise function. Unfortunately, this is not the case if we apply this technique on a highly deformed reservoir grid. In such a case, the bijectivity of the function will not be ensured and an additional step is necessary to restore the bijectivity of the mapping function.

3.2 Correction of the Mapping

Point insertion algorithm

In the case of highly deformed reservoir grids, the triangulation built in the ideal space will not be valid in the real space. Indeed, some of the elements can

be invalid which creates locally self intersections. An optimisation procedure has been proposed to restore the validity of the triangulation. The basic idea is to insert new vertices in the triangulation in order to catch the bending induced by the mapping between the two spaces. It consists on the following steps:

1. mark the invalid tetrahedra in the real space,
2. insert the barycenter of the marked tetrahedron in the delaunay triangulation, thus in the ideal space (step 1 in Figure 3),
3. compute the best position of the inserted vertices so that their associated ball of tetrahedra is valid (step 2 in Figure 3),
4. stop if no invalid tetrahedra remain, else loop again.

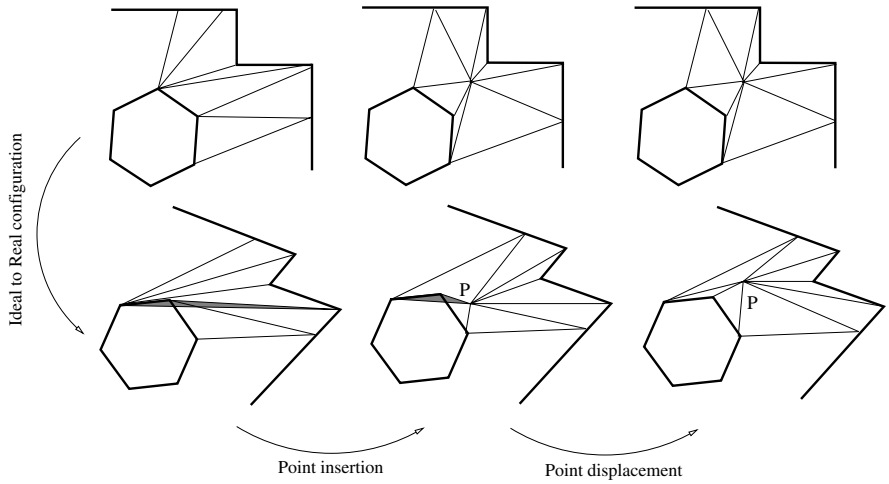

Fig. 3. Point insertion algorithm.

The insertion of vertices in the triangulation is done in the ideal space. By consequence, all tetrahedra are valid in the ideal space. On the other hand, this can not be ensured in the real space, and inverted tetrahedra can be produced. We thus have to find a position for the inserted vertex that make simultaneously all its adjacent tetrahedra non inverted when it is possible. The methodology used to compute the best position of the free vertex is discussed below.

Untangling algorithm using an objective function

The optimization loop we described involves to find an optimal position for the vertex of a ball of tetrahedra. An efficient algorithm was published in [4] which consists in minimizing the following objective function:

$$f(p_i) = \sum_{j=0}^{N_j} (|V_j| - V_j)^2 \,. \tag{4}$$

where V_j is the volume of the jth tetrahedron adjacent to vertex p_i. The convexity of such a function is provable, so that a local minimum is the global minimum of the function. Due to the absolute value, the function is not differentiable. Consequently, the function is not "smooth" and there is not an unique minimum but a *region* where the function has the same value, corresponding to the location of p_i where no tetrahedra has a null volume. If such a region does not exist, then there is a single minimum which minimizes the sum of the square of negative volumes. Nevertheless, negative volumes remain in this case.

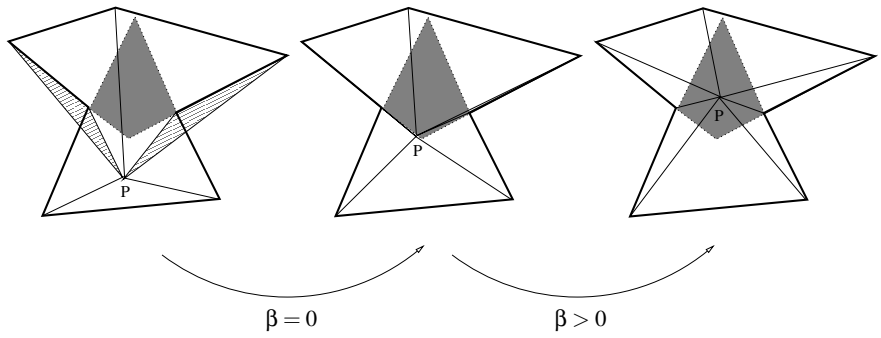

Fig. 4. Point displacement algorithm.

Minimizing the above function using a steepest descent algorithm leads to find a point on the boundary of the stationnary region. In this case, at least one tetrahedron has a null volume. This is not what was initially wanted, because tetrahedra with a null volume create a singularity in the deformation function. Our aim is thus to find the best location for the vertex p_i inside the "stationnary region" (grey area in Figure 4). To achieve such an objective, an idea is to cheat about the computed volume of the tetrahedra. Whereas the volume of a considered tetrahedron is null, a small value is subtracted to the computed volume, so that the declared volume is still negative. That is why a parameter β is introduced, which has the effect of reducing the area of the stationnary region and thus allowing the inserted vertex to "enter" in the "stationnary region". Thanks to this parameter, at convergence, there is not anymore tetrahedra with a null volume. The objective function becomes:

$$f(p_i) = \sum_{j=0}^{N_j} (|V_j - \beta| - (V_j - \beta))^2 \,. \tag{5}$$

The main difference with the algorithm presented in [4] is that our minimization includes two different steps. First, we do the computation with $\beta = 0$, then we compute the parameter β considering the local configuration. Finally, we launch again the minimization of the function with the new value of β, starting from the vertex position found at the first step.

Remark: An important point is the determination of the value of the parameter β. This parameter must be computed so that the smallest volume is maximized. If β is too small, at least one tetrahedron would keep a volume very small, whereas an other position of p_i would give a higher volume. If β is too high, the computed position can be outside the "stationnary region". The parameter β cannot be a priori defined because it depends on the configuration encountered. In practice, we compute the volume of the smallest tetrahedron which has a non null volume and associate this value (divided by the number of null volume plus one) to β. The aim is to distribute this smallest non null volume between the tetrahedra having a null volume.

As we said, the "stationnary region" we discussed could also not exist for the given configuration. In such a case, the final value of the objective function is more than zero and the point is just not moved. Thus, we loop over all free vertices and we wait until one or more vertices of a given configuration is moved. If no vertex move occurs within an entire loop, the configuration is blocked and new free vertices have to be inserted in order to allow the displacement of the blocked vertices.

The final triangulation provides us a way to transpose the polyhedral mesh (built in the ideal space) in the real space.

4 Numerical Example

We give here an example of an hybrid mesh built inside a deformed reservoir grid. The reservoir grid (see Figure 6) is made of 18000 cells and is considered to be deformed enough to be a good example of what could be encountered in general. The radial mesh is a classical grid built around the well trajectory containing 2200 cells.

Figure 7 shows in detail the radial mesh in the real space and in the intermediate space. The intermediate space generated before the ϕ correction changes the reservoir grid into a cartesian one (step O_1 in Figure 5). The right picture shows the radial mesh in the intermediate space. The initial radial characteristic of the mesh is lost.

The radial mesh is then rebuilt (step O_2) and the Delaunay triangulation of the vertices of the cavity is processed. The triangulation obtained is not valid in the real space (143 invalid tetrahedra). The vertex insertion algorithm is thus launch and after 6 insertion loops and 101 vertices finally inserted, no invalid tetrahedron remains.

The final polyhedral mesh is obtained using the cartesian methodology (step O_3) and the deformation defined by ϕ^{-1} (step O_4). Graphical results of

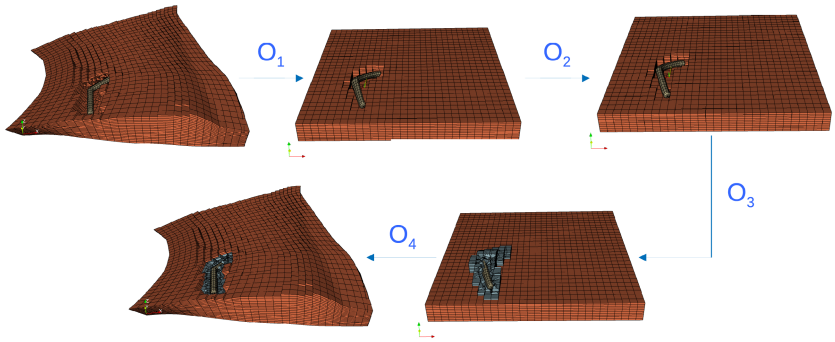

Fig. 5. Detailled description of the generation methodology.

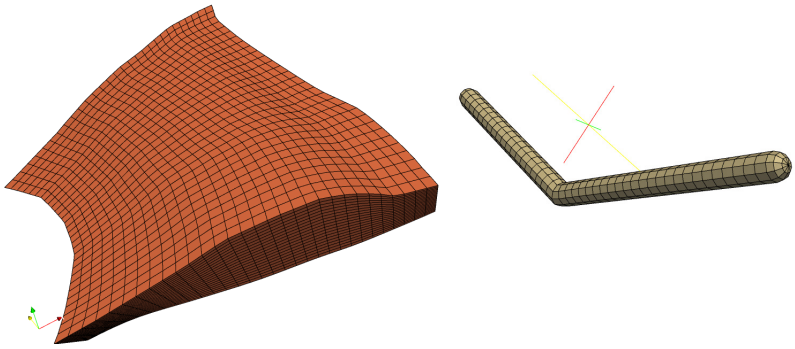

Fig. 6. Initial reservoir grid (30 × 30 × 20 cells) and radial grid around the well (5 × 10 × 44 cells).

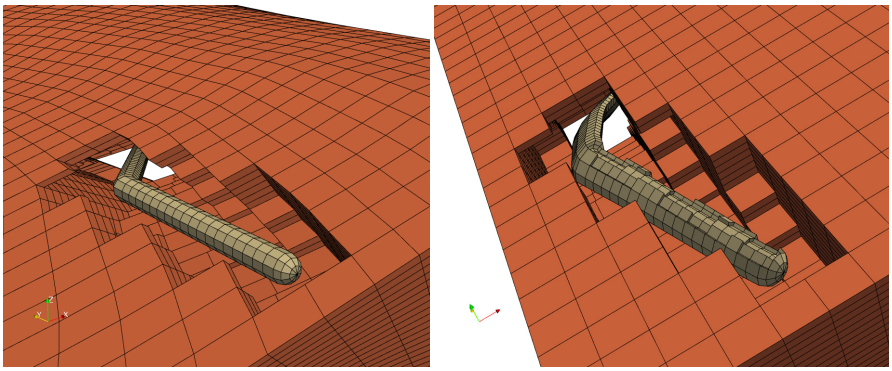

Fig. 7. Detail of the radial in the real space and in the intermediate space (cartesian configuration of the reservoir grid).

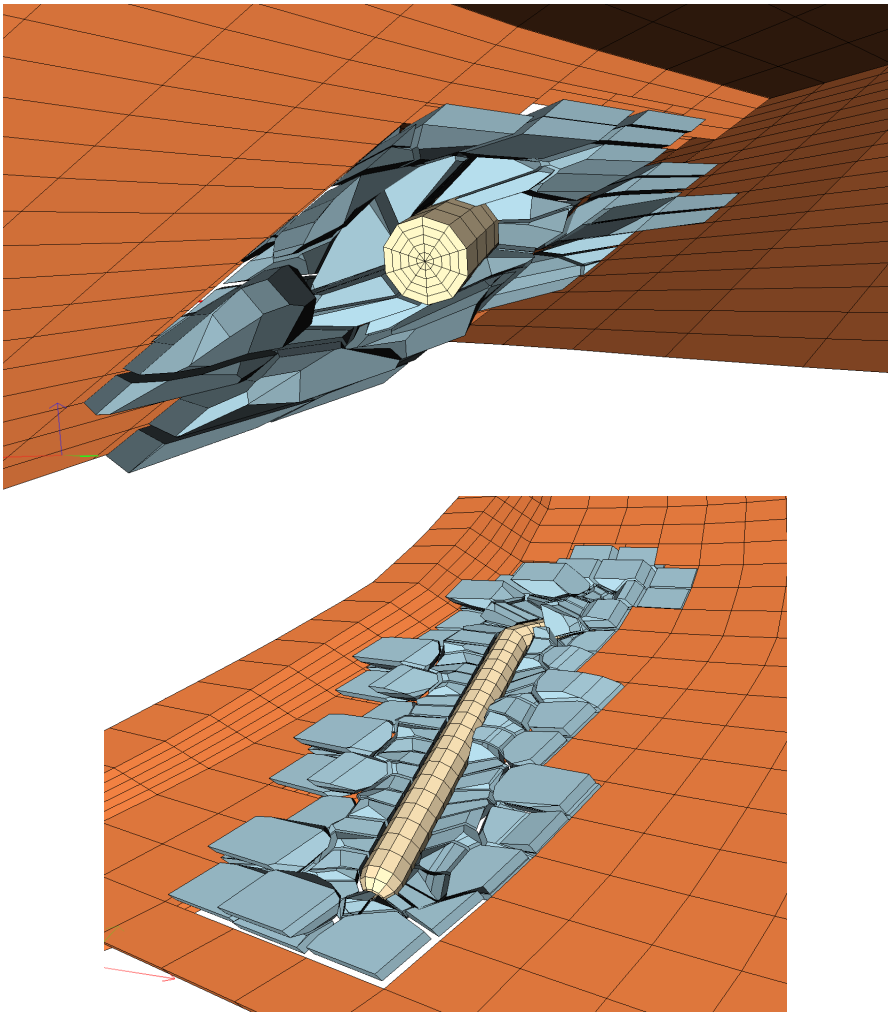

Fig. 8. Details of a hybrid mesh in the $\mathcal{E}_{real}$ real space.

the whole hybrid mesh are given in Figure 8. The generated polyhedral mesh contains 4887 vertices, 6432 faces and 950 cells.

5 Conclusions

The results obtained by the second methodology are encouraging and the hybrid mesh generation is now possible on realistic cases. However, some additional work needs to be done. The mapping could be well adapted by using, after the untangler, a smoother rather than setting a parameter depending

of the configuration. Then, this method induces a severe deformation to the polyhedral mesh which makes it invalid for simulations. A correction step by means of optimization is required. Two kinds of optimization are used:

- A priori optimization: the deformation can be quite severe locally. The aim of this optimization is to refine locally the polyhedral mesh so that the deformation is distributed over several cells. The refinement process is applied while the deformation per cell remains too high.
- A posteriori optimization: due to the fact that the faces and edges of the mesh are not suited to the simulations, a classical mesh optimization based on geometrical criterions will be applied in the $\mathcal{E}_{real}$ space.

Finally, let's not forget that we are not looking for a polyhedral mesh that has perfect faces and edges. Indeed, a power diagram owning all the given cavity faces does not exist. That is the reason why we are looking for a compromise as close as possible to a power diagram. We now have a methodology to build a conformal polyhedral mesh from surfaces which do not suit the power diagram generation methodology requirements. This new approach is very promising an simulation tests on real data are planned in order to validate the efficiency of this technology.

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