A New Technique for Quad-Dominant Adaptive Mesh Generation

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A new technique for adaptive grid generation is presented. This technique is fast and based on quad-tree decomposition. The resulting mesh is quad-dominant and ready for the application of finite element and multigrid methods. The lower bound of the interior angles of any element is set to be 45° except at those parts of the boundaries where the geometry is discontinuous. Narrow regions of the domain are automatically detected and represented efficiently. An algorithm for mesh modification in the boundary layer region is presented. Several application examples are provided to emphasize the main features of this new approach.

1 Introduction

Grid generation is a first step in the solution of many computational problems, especially those arising in scientific computing and computer graphics. For example, the accuracy of the numerical solution of partial differential equations depends on the quality of the grid used in the discretization of the problem.

Over the last two decades, adaptive methods have stirred much interest in the engineering community. For compressible flow applications, such methods are crucial because of the pressing need for accurate computation of flows with variable density and/or shock waves. Adaptive methods offer a means of tackling complex flow problems at a reasonable cost and of controlling the accuracy of numerical simulations.

The purpose of this paper is to describe a high-quality mesh generation technique for planar domains of arbitrary shape based on quad-tree decomposition. Our goal for this new technique is to achieve the following requirements:

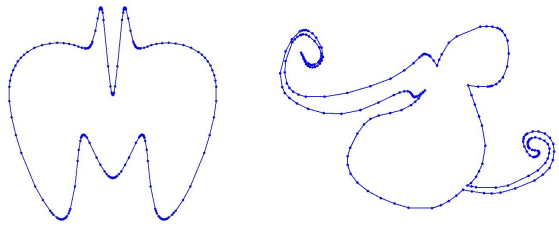
- Fast and adaptive.
- Capable of automatic detection and efficient representation of narrow and slope-discontinuous regions in the domain.
- Efficient representation of the boundary layer region.

- Ready for the application of finite element methods.
- Ready for the application of multigrid and line-solver methods.
- Most of the domain is covered by squares.
- Minimum angle in any element should be greater than or equal to 45° with the exception of those elements with a node where the boundary of the domain is slope-discontinous.

Our mesh generation technique can be summarized in the following steps:

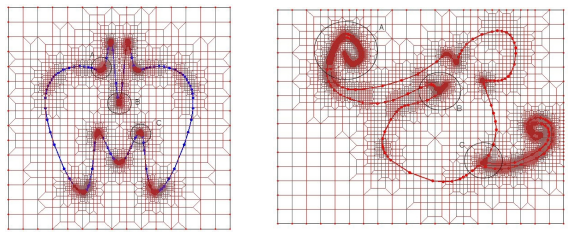
1. Linear representation of the domain boundaries based on its curvature:
   - Each closed boundary curve is represented initially with a two-line segment ploryline.
   - The number of the line segements in each polyline increases through an iteraive adaptive refinement procedure.
   - A parameter, ϵ, is responsible for the termination of the iterative refinement procedure which stops when all the angles, α_i, of the closed polyline satisfy $180^\circ - \epsilon < \alpha_i < 180^\circ + \epsilon$
2. Refinement of a background Cartesian mesh based on quad-tree decomposition and distribution of boundary edges:
   - The size of the near boundary elements is determined based on the length of the nearest boundary edge.
   - Narrow domain regions are detected and refined.
3. Detection of the points out of the domain.
   - Standard Delaunay triangulation is used to cover each closed curve with triangles.
   - Exterior elements are then removed.
4. Optimization of near-boundary elements:
   - Buffer zone creation around each shape and removal of external elements.
   - Covering the buffer zone between the terminal elements and the boundaries of the domain.
   - Modification of the boundary layer elements to enable efficient capturing of the velocity variables for viscous flows.
5. Mesh adaption based on some error function during numerical simulation.

The output of these steps is demonstrated using the following figures. Figure 1 shows the linear representation of two different domain boundaries; a smooth one with varying curvature and another one with discontinuous-slope and narrow regions. Figure 2 shows the spatial decomposition around the two types of boundaries (varying curvature and narrow region). The result from step 3 in which the domain is covered with triangles is shown in Fig. 3. The optimization step and the modification of the elements in the boundary layer region is demonstrated using Figs. 4 and 5. Finally the solution based adaptation and the multigrid levels are demonstrated in Figs. 6 and 7 for unsteady flow over two vertical cylinders..

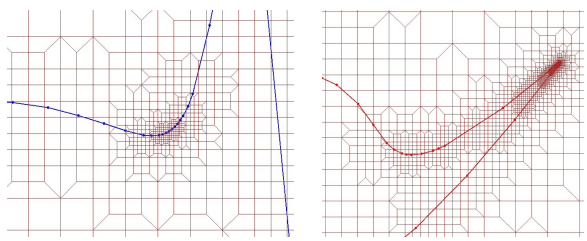


(a) $\epsilon = 10°$ (b) $\epsilon = 30°$

Fig. 1. Linear representation of the domain boundaries

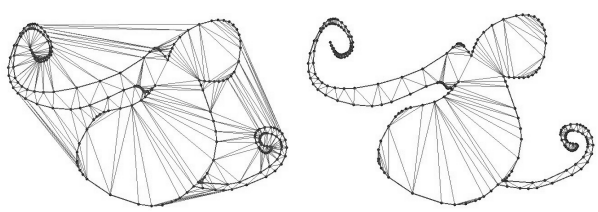


(a) Domain with boundaries of varying curvature (b) Domain with narrow regions



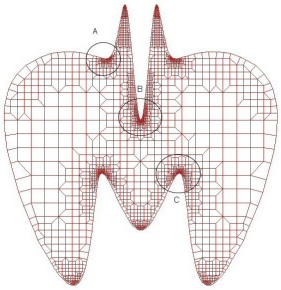
(c) Zoom A (d) Zoom C

Fig. 2. Spatial decomposition based on quad-tree and boundary edge distribution

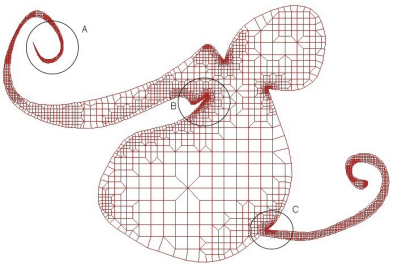


(a) Output of Standard Delaunay algorithm (b) External Elements Removed

Fig. 3. Covering the domain with triangles

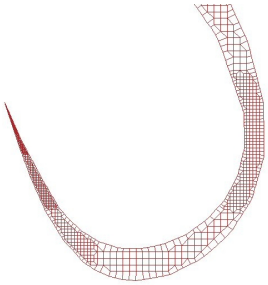


(a) Domain with boundaries of varying curvature



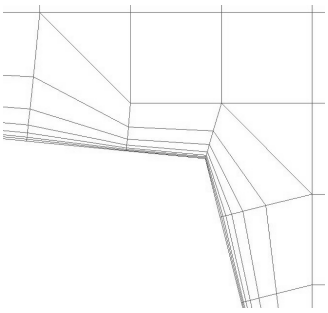
(b) Domain with narrow regions

(c) Zoom A

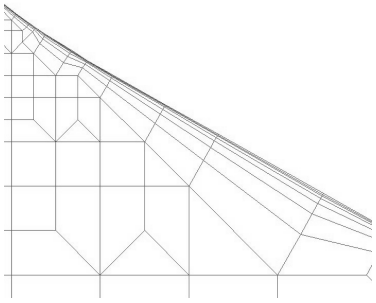


(d) Zoom A

Fig. 4. Optimization Step

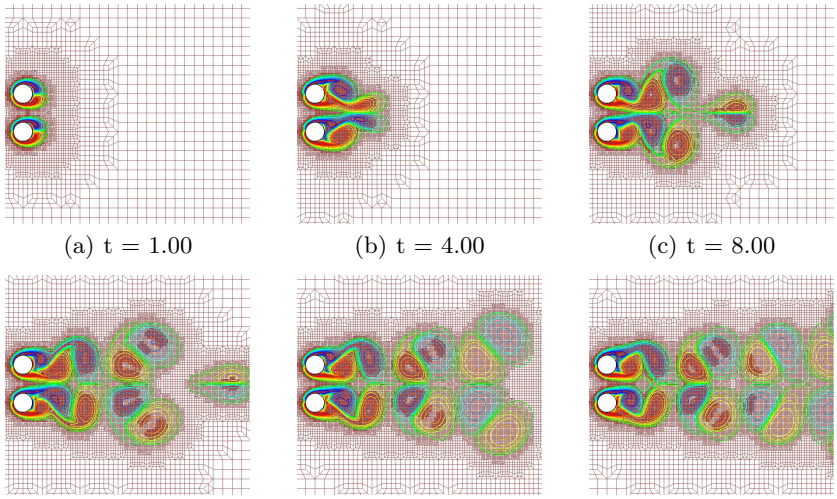


(a)



(b)

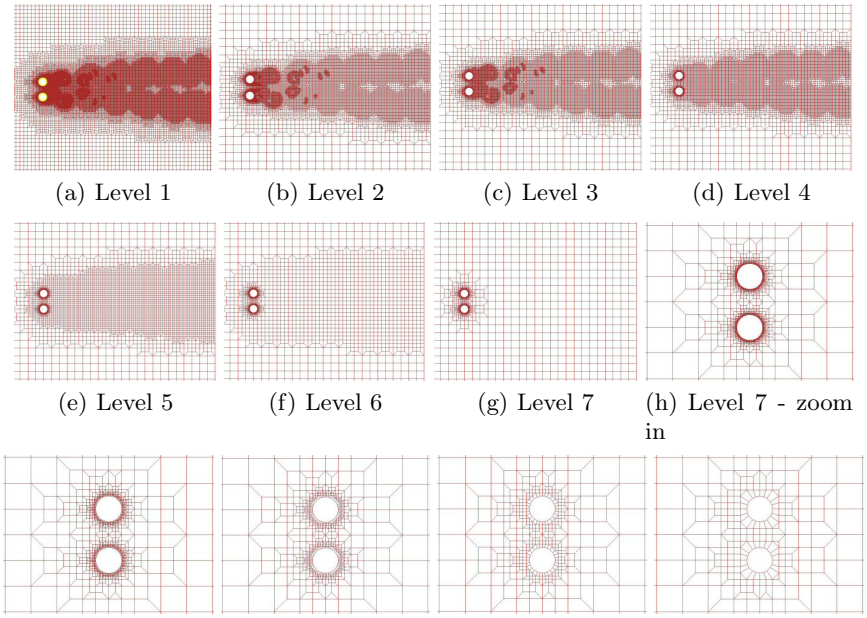
Fig. 5. Modification of the elements in the boundary layer region



(a) t = 1.00 (b) t = 4.00 (c) t = 8.00

(d) t = 12.00 (e) t = 16.00 (f) t = 20.00

Fig. 6. Evolution of grid and vorticity contours for flow over two vertical cylinders at $Re = 200$



(a) Level 1 (b) Level 2 (c) Level 3 (d) Level 4

(e) Level 5 (f) Level 6 (g) Level 7 (h) Level 7 - zoom in

(i) Level 8 (j) Level 9 (k) Level 10 (l) Level 11

Fig. 7. Grid Levels for a domain with two vertical cylinders