Quadrilateral Mesh Improvement

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1 Introduction

In this paper, we build on the mesh improvement methods intended for parallel execution with the paving method [1] and introduce topological changes in an existing quadrilateral-only mesh to improve element shape and nodal valence. This discussion is intended to demonstrate the effectiveness of utilizing mesh cleanup operations as a post-processing step to other mesh generation algorithms. The technique is applied to meshes produced via triangle-to-quadrilateral subdivision and octree-based surface generation methods to exemplify dramatric improvement in mesh quality independent of the original mesh structure and generating method.

2 Mesh Improvement Algorithm

2.1 Mesh Improvement

The purpose of this research is to identify poor topological conditions in quadrilateral meshes and replace specific templates with alternate templates to improve quality and topology and improve mesh structure by targeting high vertex valence. The templates are illustrated in Figure 1. Although most *Doublets* are resolved with a face close operation, doublets that are constrained by curves must be treated with a doublet insertion and edge swap (case 2) or a face open operation (case 3). *Triangular Quadrilaterals* are cleaned up by a doublet insertion and then are treated by case 2 of the *Doublet Removal* operation. Similarly, *Flat Quadrilaterals* are modified with an edge swap and then treated as *Triangular Quadrilaterals*. The *Diamond Collapse* and *Quadrilateral Edge Swap* operations are respectively a face close operation or an edge swap operation performed on specific topologies, always resulting in a more structured mesh. *High Valence Nodes* are resolved with either a face open

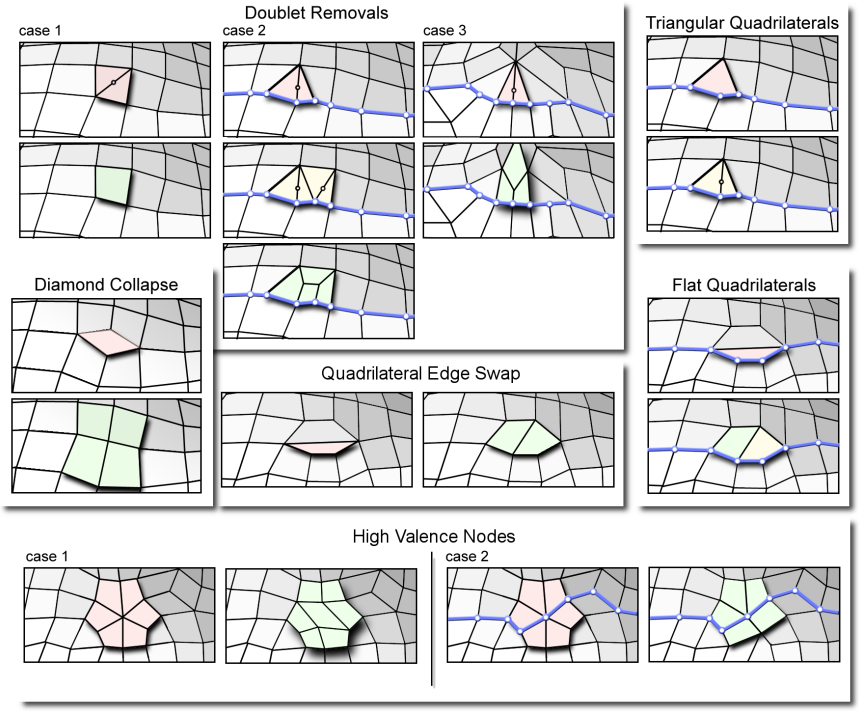

Fig. 1. Quadrilateral improvement operations.

operation (case 1) or a face close operation (case 2). To avoid creating additional problematic topologies, the more simple operations are performed first and the more difficult ones later, in this order: *Diamond Removal, Doublet Removal, Quadrilateral Edge Swap, Triangle Quads, and finally High Valence Nodes.*

2.2 Geometric Constraints and Smoothing

When performing local topological changes to a surface mesh, care must be taken not to alter the geometry of the model. Geometric features are identified a priori and template replacement that results in the removal of geometric features is disallowed. Additionally, to preserve geometric characteristics that are not explicitly defined by curves or vertices, the normal of each pair of adjacent quadrilaterals is calculated. Large differences in the angles of the normal vectors of adjacent quadrilaterals indicate the presence of a geometric feature. If this is the case, the edge and nodes common to the quadrilaterals are marked to be treated as a curve to preserve the feature.

Element quality is considered in addition to local topology, and smoothing is performed after each iteration of the algorithm. An iterative centroid area

scheme is used because it is very straight-forward to implement and requires much less computational effort than many other smoothers [2].

3 Examples

3.1 Plane Mesh Improvement

The first example, illustrated in Figure 2, is a triangle mesh that has been subdivided into quadrilaterals using a Catmull-Clark subdivision and then cleaned up with the method presented in Section 2. As can be seen in Table 1 the number of elements was reduced and the mesh quality was improved significantly, with dramatic improvement in the mesh topology as shown in Figure 2.

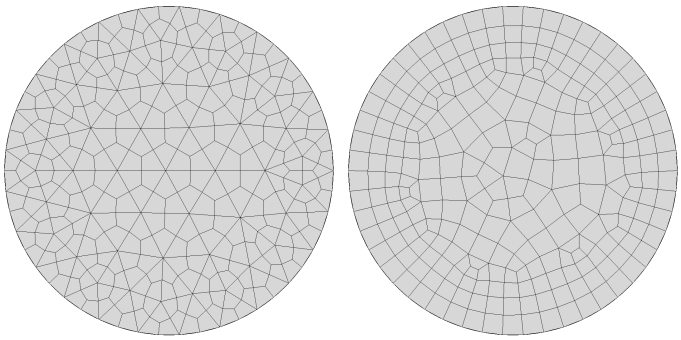

Fig. 2. Before and after quadrilateral improvement on a triangle mesh that has been split into quadrilaterals.

3.2 Surface Mesh Improvement

The triceratops and rabbit examples demonstrate the effectiveness of the algorithm on three-dimensional surface meshes, illustrated in Figure 3. The topology results are illustrated in Figure 3 and the element quality and node valences are further analyzed in Table 2. The cleanup algorithm presented in this paper improves the average scaled Jacobian of the meshes by significantly increasing the number of structured nodes and generating many near square elements.

3.3 Limitations

The methods demonstrated in this paper are straight-forward to implement and can be applied to any quadrilateral mesh. Since the improvement operations tend to increase structure, the likelihood of algorithmic circularity is

Table 1. Before and after comparisons of the number of quadrilateral elements, the average, minimum and maximum scaled Jacobians (SJ), and the valence histograms (number of nodes |N| and % of the total) of mesh shown in Figure 2.

Quality Metric	Circular Disk				
	Before		After		% Change
|Quad|	333		269		−19.2
Ave. SJ	0.804		0.951		18.2
Min. SJ	0.648		0.717		10.7
Max. SJ	0.878		1.000		13.8
Histogram	|N|	%	|N|	%	
Val. 2	0	0.0	0	0.0	
Val. 3	111	30.9	26	8.8	
Val. 4	179	49.9	222	75.3	
Val. 5	13	3.6	47	15.9	
Val. 6	49	13.6	0	0	
Val. 7^+	7	1.9	0	0	

Table 2. Before and after comparisons of the number of quadrilateral elements, the average, minimum and maximum scaled Jacobians (SJ), and the valence histograms (number of nodes |N| and % of the total) of the two meshes shown in Figure 3.

Quality Metric	Rabbit					Triceratops				
	Before		After		% Change	Before		After		% Change
|Quad|	21714		19675		−9.4	19274		17607		−8.6
Ave. SJ	0.904		0.968		7.1	0.903		0.962		6.5
Min. SJ	0.453		0.436		−3.8	−0.500		0.031		106.2
Max. SJ	1.000		1.000		0.0	1.000		1.000		0.0
Histogram	|N|	%	|N|	%		|N|	%	|N|	%	
Val. 2	0	0.0	0	0.0		0	0.0	0	0.0	
Val. 3	5099	23.5	1078	5.5		4190	21.7	880	5.0	
Val. 4	12163	56.0	17530	89.1		11434	59.3	15859	90.1	
Val. 5	3817	17.6	1068	5.4		3122	16.2	868	4.9	
Val. 6	637	2.9	1	0.0		530	2.7	2	0.0	
Val. 7^+	0	0.0	0	0.0		0	0.0	0	0.0	

low. The order of operations can affect the level of results achieved and the heuristic ordering that we have chosen for our algorithm may not be generally appropriate for all mesh generation methods. However, for the examples shown, this recipe has led to pleasing results, especially for the triangle subdivision mesh.

4 Conclusion

In this paper, we introduce topological changes in an existing quadrilateral mesh to improve quadrilateral shape and nodal valence. Our purpose is to

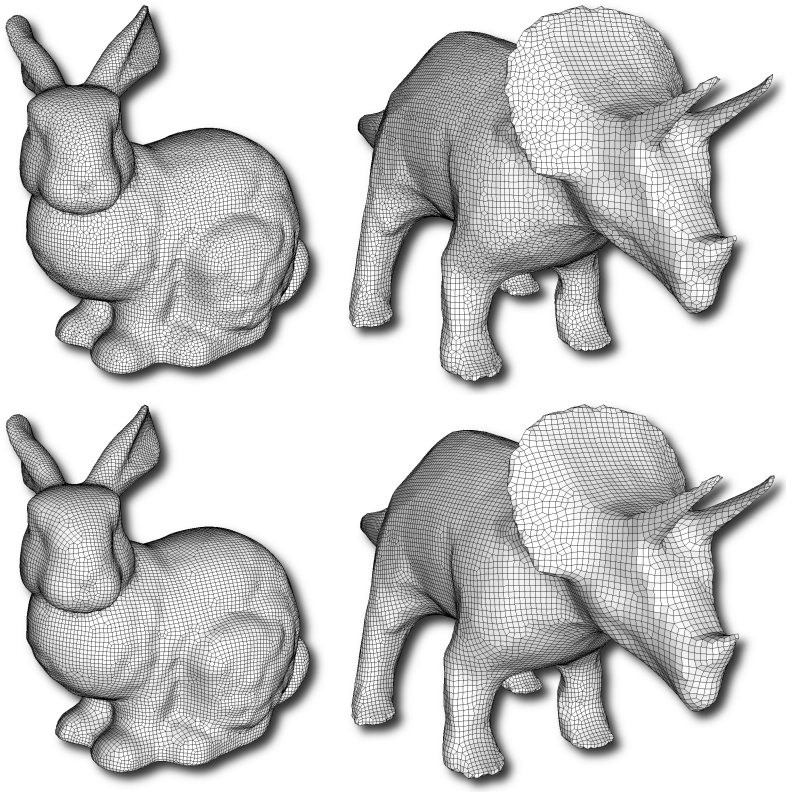

Fig. 3. Before and after quadrilateral improvement on two meshes constructed as grafted quadrilateral surfaces. Despite the generation type and original connectivity, our improvement methods greatly increase the percentage of ideal nodes and element quality.

show the impact of topological improvement operations for all quadrilateral generation methods. We have demonstrated these mesh improvement operations with several different quadrilateral mesh schemes and have shown dramatic improvement in the quality of these meshes.

References

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