Unstructured Mesh Related Issues In Computational Fluid Dynamics (CFD) – Based Analysis And Design

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Overview

• History and current state of unstructured grid technology of CFD
  – Influence of grid generation technology
  – Influence of solver technology
• Examples of unstructured mesh CFD capabilities
• Areas of current research
  – Adaptive mesh refinement
  – Moving meshes
  – Overlapping meshes
  – Requirements for design methods
  – Implications for higher-order accurate Discretizations
CFD Perspective on Meshing Technology

• CFD initiated in structured grid context
  – Transfinite interpolation
  – Elliptic grid generation
  – Hyperbolic grid generation
• Smooth, orthogonal structured grids
• Relatively simple geometries
CFD Perspective on Meshing Technology

- Evolved to Sophisticated Multiblock and Overlapping Structured Grid Techniques for Complex Geometries

Overlapping grid system on space shuttle (Slotnick, Kandula and Buning 1994)
CFD Perspective on Meshing Technology

• Unstructured meshes initially confined to FE community
  – CFD Discretizations based on directional splitting
  – Line relaxation (ADI) solvers
  – Structured Multigrid solvers

• Sparse matrix methods not competitive
  – Memory limitations
  – Non-linear nature of problems
Current State of Unstructured Mesh CFD Technology

• Method of choice for many commercial CFD vendors
  – Fluent, StarCD, CFD++, …

• Advantages
  – Complex geometries
  – Adaptivity
  – Parallelizability

• Enabling factors
  – Maturing grid generation technology
  – Better Discretizations and solvers
Maturing Unstructured Grid Generation Technology (1990-2000)

• Isotropic tetrahedral grid generation
  – Delaunay point insertion algorithms
  – Surface recovery
  – Advancing front techniques
  – Octree methods

• Mature technology
  – Numerous available commercial packages
  – Remaining issues
    • Grid quality
    • Robustness
    • Links to CAD
Maturing Unstructured Grid Generation Technology (1990-2000)

• Anisotropic unstructured grid generation
  – External aerodynamics
    • Boundary layers, wakes: $O(10^{**4})$
  – Mapped Delaunay triangulations
  – Min-max triangulations
  – Hybrid methods
    • Advancing layers
    • Mixed prismatic – tetrahedral meshes
Anisotropic Unstructured Grid Generation

- Hybrid methods
  - Semi-structured nature
  - Less mature: issues
    - Concave regions
    - Neighboring boundaries
    - Conflicting resolution
    - Conflicting Stretchings

VGRIDns Advancing Layers
c/o S. Pirzadeh, NASA Langley

• Edge-based data structure
  – Building block for all element types
  – Reduces memory requirements
  – Minimizes indirect addressing / gather-scatter
  – Graph of grid = Discretization stencil
    • Implications for solvers, Partitioners

• Multigrid solvers
  – Multigrid techniques enable optimal $O(N)$ solution complexity
  – Based on sequence of coarse and fine meshes
  – Originally developed for structured grids

- Agglomeration Multigrid solvers for unstructured meshes
  - Coarse level meshes constructed by agglomerating fine grid cells/equations
Agglomeration Multigrid

- Automated Graph-Based Coarsening Algorithm
- Coarse Levels are Graphs
- Coarse Level Operator by Galerkin Projection
- Grid independent convergence rates (order of magnitude improvement)
Enabling CFD Solver Developments

- Line solvers for Anisotropic problems
  - Lines constructed in mesh using weighted graph algorithm
  - Strong connections assigned large graph weight
  - (Block) Tridiagonal line solver similar to structured grids

• Graph-based Partitioners for parallel load balancing
  – Metis, Chaco, Jostle
• Edge-data structure → graph of grid
• Agglomeration Multigrid levels = graphs
• Excellent load balancing up to 1000’s of processors
  – Homogeneous data-structures
  – (Versus multi-block / overlapping structured grids)
Practical Examples

- VGRIDns tetrahedral grid generator
- NSU3D Multigrid flow solver
  - Large scale massively parallel case
  - Fast turnaround medium size problem
NASA Langley Energy Efficient Transport

• Complex geometry
  – Wing-body, slat, double slotted flaps, cutouts

• Experimental data from Langley 14x22ft wind tunnel
  – Mach = 0.2, Reynolds=1.6 million
  – Range of incidences: -4 to 24 degrees
Initial Mesh Generation (VGRIDns)
S. Pirzadeh, NASA Langley

• Combined advancing layers- advancing front
  – Advancing layers: thin elements at walls
  – Advancing front: isotropic elements elsewhere
• Automatic switching from AL to AF based on:
  – Cell aspect ratio
  – Proximity of boundaries of other fronts
  – Variable height for advancing layers
• Background Cartesian grid for smooth spacing control
• Spanwise stretching
  – Factor of 3 reduction in grid size
VGRID Tetrahedral Mesh

- 3.1 million vertices, 18.2 million tets, 115,489 surface pts
- Normal spacing: 1.35E-06 chords, growth factor=1.3
Prism Merging Operation

- Combine Tetrahedra triplets in advancing-layers region into prisms
  - Prisms entail lower complexity for solver
- VGRIDns identifies originating boundary point for ALR vertices
  - Used to identify candidate elements
  - Pyramids required as transitional elements
Prism Merging Operation

- Initial mesh: 18.2M Tetrahedra
- Merged mesh: 3.9M prisms, 6.6M Tets, 47K pyramids
  - 64% of Tetrahedra merged
Global Mesh Refinement

• High-resolution meshes require large parallel machines
• Parallel mesh generation difficult
  – Complicated logic
  – Access to commercial preprocessing, CAD tools
• Current approach
  – Generate coarse (O(10**6) vertices on workstation
  – Refine on supercomputer
Global Mesh Refinement

- Refinement achieved by element subdivision
- Global refinement: 8:1 increase in resolution
- In-Situ approach obviates large file transfers
- Initial mesh: 3.1 million vertices
  - 3.9M prisms, 6.6M Tets, 47K pyramids
- Refined mesh: 24.7 million vertices
  - 31M prisms, 53M Tets, 281K pyramids
  - Refinement operation: 10 Gbytes, 30 minutes sequentially
NSU3D Unstructured Mesh Navier-Stokes Solver

- Mixed element grids
  - Tetrahedra, prisms, pyramids, hexahedra
- Edge data-structure
- Line solver in BL regions near walls
- Agglomeration Multigrid acceleration
- Newton Krylov (GMRES) acceleration option
- Spalart-Allmaras 1 equation turbulence model
Parallel Implementation

- Domain decomposition with OpenMP/MPI communication
  - OpenMP on shared memory architectures
  - MPI on distributed memory architectures
  - Hybrid capability for clusters of SMPs
- Weighted graph partitioning (Metis)
- Coarse and fine MG levels partitioned independently
Computed Pressure Contours on Coarse Grid

- Mach=0.2, Incidence=10 degrees, Re=1.6M
Computed Versus Experimental Results

- Good drag prediction
- Discrepancies near stall
Multigrid Convergence History

- Mesh independent property of Multigrid
- GMRES effective but requires extra memory
Parallel Scalability

- Good overall Multigrid scalability
  - Increased communication due to coarse grid levels
  - Single grid solution impractical (>100 times slower)
- 1 hour solution time on 1450 PEs

<table>
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<th>No. of Procs</th>
<th>Time/Cyc</th>
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<td>1450</td>
<td>7.54</td>
<td>82.0</td>
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AIAA Drag Prediction Workshop (2001)

- Transonic wing-body configuration
- Typical cases required for design study
  - Matrix of mach and CL values
  - Grid resolution study
- Follow on with engine effects (2003)
Cases Run

- Baseline grid: 1.6 million points
  - Full drag polars for Mach=0.5,0.6,0.7,0.75,0.76,0.77,0.78,0.8
  - Total = 72 cases
- Medium grid: 3 million points
  - Full drag polar for each mach number
  - Total = 48 cases
- Fine grid: 13 million points
  - Drag polar at mach=0.75
  - Total = 7 cases
Sample Solution (1.65M Pts)

- Mach=0.75, CL=0.6, Re=3M
- 2.5 hours on 16 Pentium IV 1.7GHz
Drag Polar at Mach = 0.75

- Grid resolution study
- Good comparison with experimental data

Note: Wind tunnel data follow prescribed trip pattern; CFD data are fully turbulent
Cases Run on ICASE Cluster

- 120 Cases (excluding finest grid)
- About 1 week to compute all cases
Current and Future Issues

- Adaptive mesh refinement
- Moving geometry and mesh motion
- Moving geometry and overlapping meshes
- Requirements for gradient-based design
- Implications for higher-order Discretizations
Adaptive Meshing

• Potential for large savings through optimized mesh resolution
  – Well suited for problems with large range of scales
  – Possibility of error estimation / control
  – Requires tight CAD coupling (surface pts)
• Mechanics of mesh adaptation
• Refinement criteria and error estimation
Mechanics of Adaptive Meshing

- Various well know isotropic mesh methods
  - Mesh movement
    - Spring analogy
    - Linear elasticity
  - Local Remeshing
  - Delaunay point insertion/Retriangulation
  - Edge-face swapping
  - Element subdivision
    - Mixed elements (non-simplicial)
    - Anisotropic subdivision required in transition regions
Subdivision Types for Tetrahedra

1:2

1:4

1:8

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Subdivision Types for Prisms
Subdivision Types for Pyramids

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Subdivision Types for Hexahedra
Adaptive Tetrahedral Mesh by Subdivision
Adaptive Hexahedral Mesh by Subdivision
Adaptive Hybrid Mesh by Subdivision

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Anisotropic Adaptation Methods

• Large potential savings for 1 or 2D features
  – Directional subdivision
    • Assumes element faces to line up with flow features
    • Combine with mesh motion
  – Mapping techniques
    • Hessian based
    • Grid quality
Refinement Criteria

• Weakest link of adaptive meshing methods
  – Obvious for strong features
  – Difficult for non-local (ie. Convective) features
    • eg. Wakes
  – Analysis assumes in asymptotic error convergence region
    • Gradient based criteria
    • Empirical criteria

• Effect of variable discretization error in design studies, parameter sweeps
Adjoint-based Error Prediction

• Compute sensitivity of global cost function to local spatial grid resolution
• Key on important output, ignore other features
  – Error in engineering output, not discretization error
    • e.g. Lift, drag, or sonic boom …
• Captures non-local behavior of error
  – Global effect of local resolution
• Requires solution of adjoint equations
  – Adjoint techniques used for design optimization
Adjoint-based Mesh Adaptation Criteria
Reproduced from Venditti and Darmofal (MIT, 2002)
Adjoint-based Mesh Adaptation Criteria
Reproduced from Venditti and Darmofal (MIT, 2002)

Drag Error
(Bold: Error After Correct)

$Re = 5000 \quad M_\infty = 0.5 \quad \alpha = 3^\circ$

1. Adaptation based on left-element drag
   - Nodes: 3015
   - Left: 0.3%
   - Right: 27.0%
   - Total: 13.7%

2. Adaptation based on right-element drag
   - Nodes: 4526
   - Left: 3.7%
   - Right: 0.3%
   - Total: 2.6%

3. Adaptation based on total drag (both elements)
   - Nodes: 9758
   - Left: 0.3%
   - Right: 0.6%
   - Total: 0.0%

4. Pure Hessian-based adaptation
   - Nodes: 10561
   - Left: 4.2%
   - Right: 7.5%
   - Total: 5.8%
Adjoint-based Mesh Adaptation Criteria

Reproduced from Venditti and Darmofal (MIT, 2002)
Overlapping Unstructured Meshes

• Alternative to moving mesh for large scale relative geometry motion
• Multiple overlapping meshes treated as single data-structure
  – Dynamic determination of active/inactive/ghost cells
• Advantages for parallel computing
  – Obviates dynamic load rebalancing required with mesh motion techniques
  – Intergrid communication must be dynamically recomputed and rebalanced
    • Concept of Rendez-vous grid (Plimpton and Hendrickson)
Overlapping Unstructured Meshes

- Simple 2D transient example
Gradient-based Design Optimization

• Minimize Cost Function $F$ with respect to design variables $v$, subject to constraint $R(w) = 0$
  – $F = \text{drag, weight, cost}$
  – $v = \text{shape parameters}$
  – $w = \text{Flow variables}$
  – $R(w) = 0 \rightarrow \text{Governing Flow Equations}$

• Gradient Based Methods approach minimum along direction: $-\frac{\partial F}{\partial v}$
Grid Related Issues for Gradient-based Design

- Parametrization of CAD surfaces
- Consistency across disciplines
  - eg. CFD, structures,…
- Surface grid motion
- Interior grid motion
- Grid sensitivities
- Automation / Parallelization
Preliminary Design Geometry
X34 CAD Model

23,555 curves and surfaces

c/o J. Samareh, NASA Langley
Launch Vehicle Shape Parameterization

c/o J. Samareh, NASA Langley
Sensitivity Analysis

objective function
(e.g., Stress, $C_D$)

$v$ design variables
(e.g., span, camber)

\[
\begin{align*}
\frac{\partial F}{\partial v} &= \frac{\partial F}{\partial \text{Grid}_f} \frac{\partial \text{Grid}_f}{\partial \text{Grid}_s} \frac{\partial \text{Grid}_s}{\partial \text{Geometry}} \frac{\partial \text{Geometry}}{\partial \vec{v}} \\
&= \frac{\partial F}{\partial \text{Grid}_f} \frac{\partial \text{Grid}_s}{\partial \text{Grid}_f} \frac{\partial \text{Geometry}}{\partial \text{Grid}_s} \frac{\partial \text{Grid}_f}{\partial \vec{v}}
\end{align*}
\]

• Manual differentiation
• Automatic differentiation tools (e.g., ADIFOR and ADIC)
• Complex variables
• Finite-difference approximations

c/o J. Samareh, NASA Langley
Finite-Difference Approximation Error for Sensitivity Derivatives

Parameterized HSCT Model

% Error

Scaled Step Size

c/o J. Samareh, NASA Langley
Grid Sensitivities

$$\frac{\partial \text{Grid}_f}{\partial \mathbf{V}} = \frac{\partial \text{Grid}_f}{\partial \text{Grid}_s} \times \frac{\partial \text{Grid}_s}{\partial \text{Geometry}} \times \frac{\partial \text{Geometry}}{\partial \mathbf{V}}$$

- Ideally should be available from grid/cad software
  - Analytical formulation most desirable
  - Burden on grid / CAD software
  - Discontinuous operations present extra challenges
    - Face-edge swapping
    - Point addition / removal
    - Mesh regeneration
High-Order Accurate Discretizations

- Uniform X2 refinement of 3D mesh:
  - Work increase = \textit{factor of 8}
  - 2\textsuperscript{nd} order accurate method: \textit{accuracy increase} = 4
  - 4\textsuperscript{th} order accurate method: \textit{accuracy increase} = 16
  - For smooth solutions

- Potential for large efficiency gains
  - Spectral element methods
  - Discontinuous Galerkin (DG)
  - Streamwise Upwind Petrov Galerkin (SUPG)
Higher-Order Accurate Discretizations

- Transfers burden from grid generation to Discretization
Spectral Element Solution of Maxwell’s Equations

J. Hesthaven and T. Warburton (Brown University)
High-Order Discretizations

- Require more complete surface definition
- Curved surface elements
  - Additional element points
  - Surface definition (for high p)
Combined H-P Refinement

- Adaptive meshing (h-ref) yields constant factor improvement
  - After error equidistribution, no further benefit
- Order refinement (p-ref) yields asymptotic improvement
  - Only for smooth functions
  - Ineffective for inadequate h-resolution of feature
  - Cannot treat shocks
- H-P refinement optimal (exponential convergence)
  - Requires accurate CAD surface representation
Conclusions

• Unstructured mesh CFD has come of age
  – Combined advances in grid and solver technology
  – Inviscid flow analysis (isotropic grids) mature
  – Viscous flow analysis competitive
• Complex geometry handling facilitated
• Adaptive meshing potential not fully exploited
• Additional considerations in future
  – Design methodologies
  – New discretizations
  – New solution techniques
  – H-P Refinement