Unstructured Mesh Related Issues In Computational Fluid Dynamics (CFD) – Based Analysis And Design

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Overview

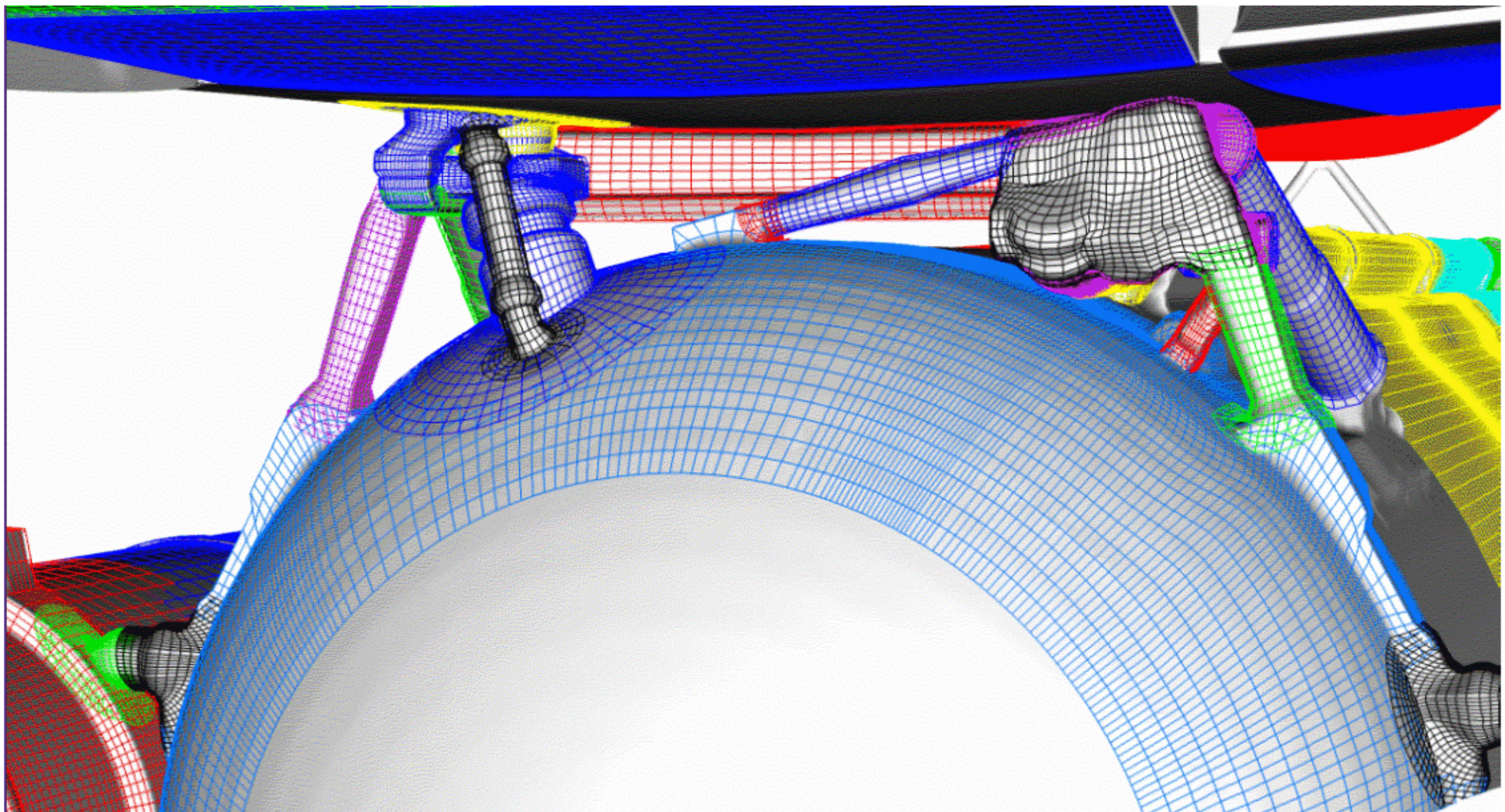
- History and current state of unstructured grid technology of CFD
  - Influence of grid generation technology
  - Influence of solver technology
- Examples of unstructured mesh CFD capabilities
- Areas of current research
  - Adaptive mesh refinement
  - Moving meshes
  - Overlapping meshes
  - Requirements for design methods
  - Implications for higher-order accurate Discretizations

CFD Perspective on Meshing Technology

- CFD initiated in structured grid context
  - Transfinite interpolation
  - Elliptic grid generation
  - Hyperbolic grid generation
- Smooth, orthogonal structured grids
- Relatively simple geometries

CFD Perspective on Meshing Technology

- Evolved to Sophisticated Multiblock and Overlapping Structured Grid Techniques for Complex Geometries



Overlapping grid system on space shuttle (Slotnick, Kandula and Buning 1994)

CFD Perspective on Meshing Technology

- Unstructured meshes initially confined to FE community
  - CFD Discretizations based on directional splitting
  - Line relaxation (ADI) solvers
  - Structured Multigrid solvers
- Sparse matrix methods not competitive
  - Memory limitations
  - Non-linear nature of problems

Current State of Unstructured Mesh CFD Technology

- Method of choice for many commercial CFD vendors
  - Fluent, StarCD, CFD++, …
- Advantages
  - Complex geometries ←
  - Adaptivity
  - Parallelizability
- Enabling factors
  - Maturing grid generation technology
  - Better Discretizations and solvers

Maturing Unstructured Grid Generation Technology (1990-2000)

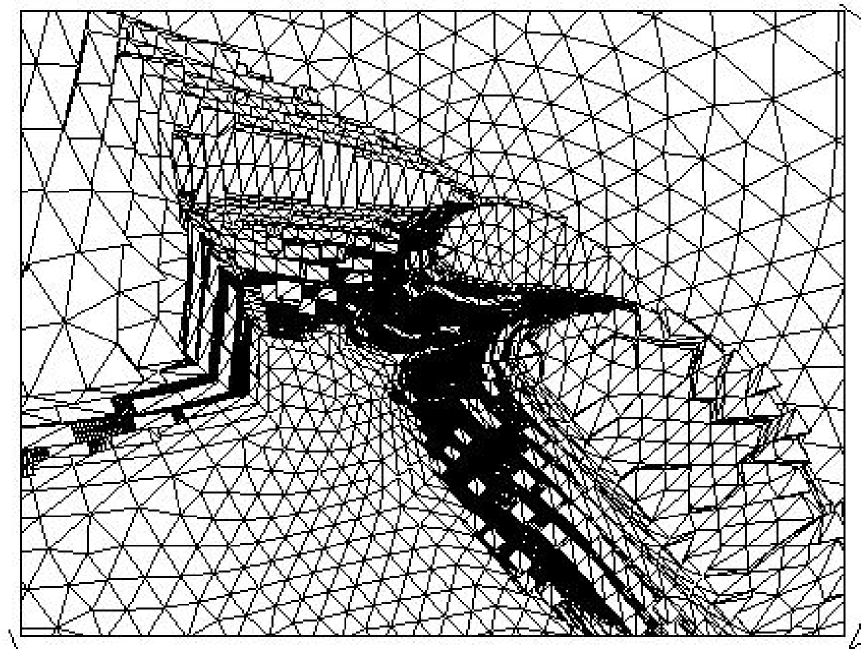
- Isotropic tetrahedral grid generation
  - Delaunay point insertion algorithms
  - Surface recovery
  - Advancing front techniques
  - Octree methods
- Mature technology
  - Numerous available commercial packages
  - Remaining issues
    - Grid quality
    - Robustness
    - Links to CAD

Maturing Unstructured Grid Generation Technology (1990-2000)

- Anisotropic unstructured grid generation
  - External aerodynamics
    - Boundary layers, wakes: O(10**4)
  - Mapped Delaunay triangulations
  - Min-max triangulations
  - Hybrid methods ←
    - Advancing layers
    - Mixed prismatic – tetrahedral meshes

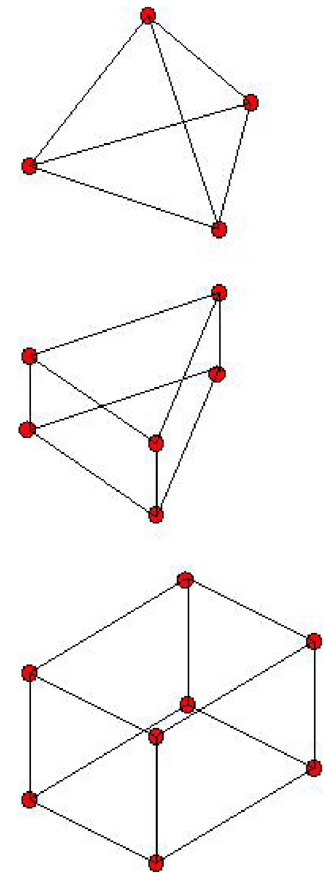
Anisotropic Unstructured Grid Generation

- Hybrid methods
  - Semi-structured nature
  - Less mature: issues
    - Concave regions
    - Neighboring boundaries
    - Conflicting resolution
    - Conflicting Stretchings

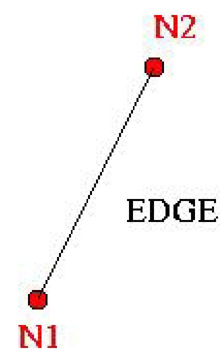


VGRIDns Advancing Layers
c/o S. Pirzadeh, NASA Langley

Enabling CFD Solver Developments (1990 – 2000)

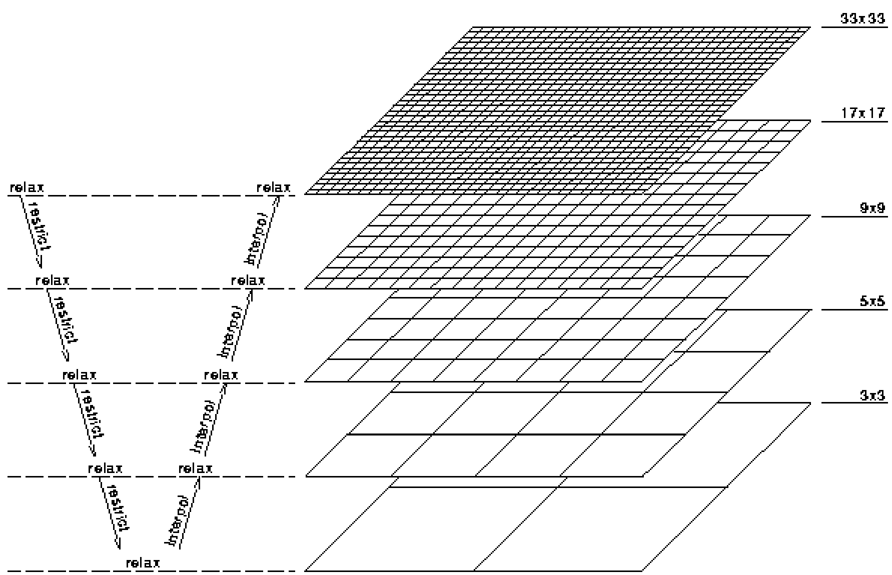


- Edge-based data structure
  - Building block for all element types
  - Reduces memory requirements
  - Minimizes indirect addressing / gather-scatter
  - Graph of grid = Discretization stencil
    - Implications for solvers, Partitioners



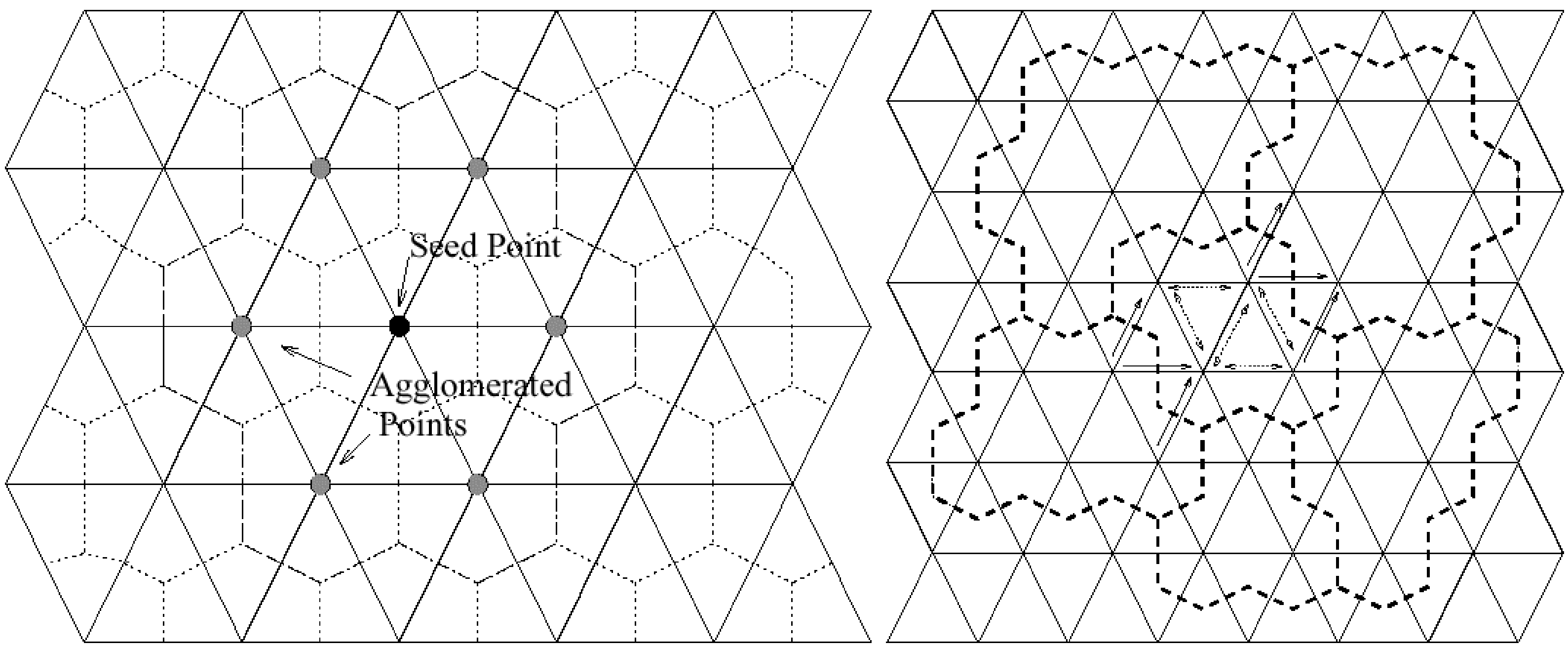
Enabling CFD Solver Developments (1990 –2000)

- Multigrid solvers
  - Multigrid techniques enable optimal O(N) solution complexity
  - Based on sequence of coarse and fine meshes
  - Originally developed for structured grids

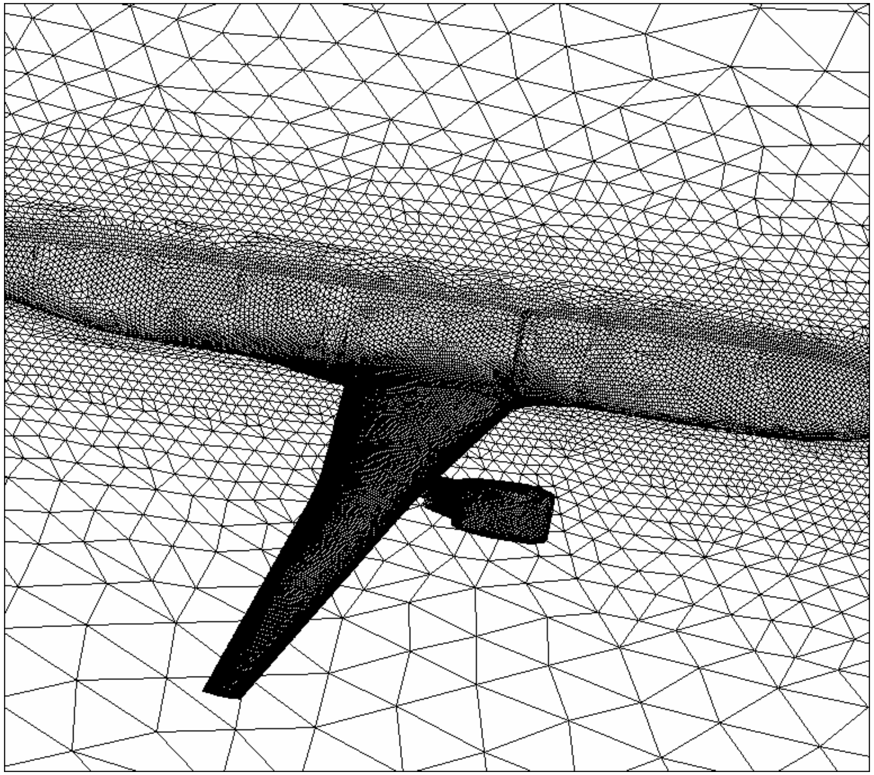


Enabling CFD Solver Developments (1990 –2000)

- Agglomeration Multigrid solvers for unstructured meshes
  - Coarse level meshes constructed by agglomerating fine grid cells/equations



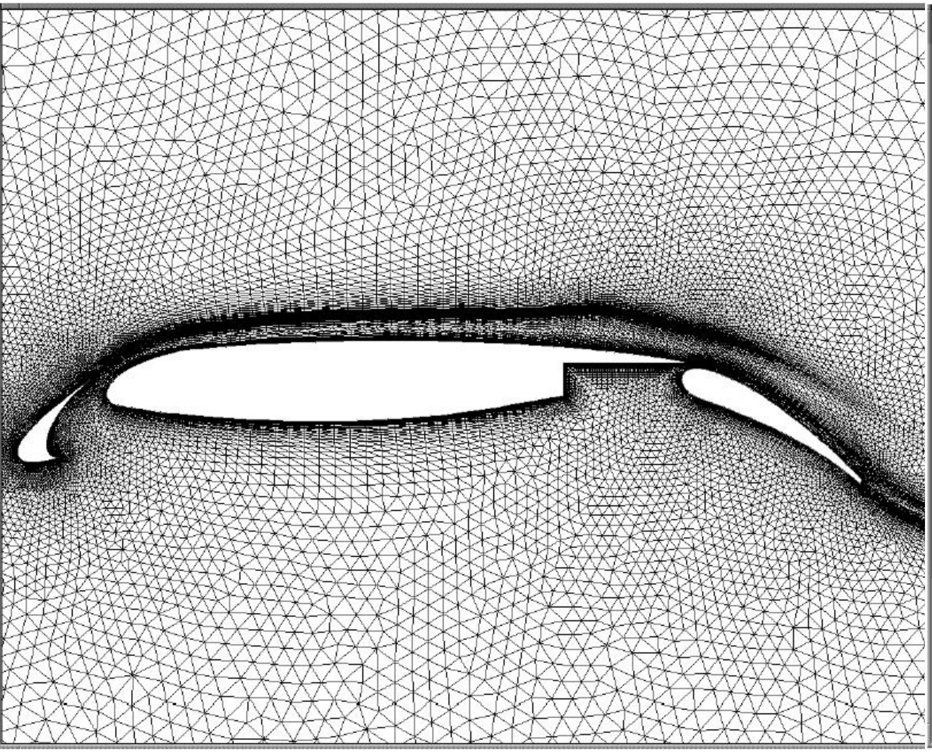
Agglomeration Multigrid

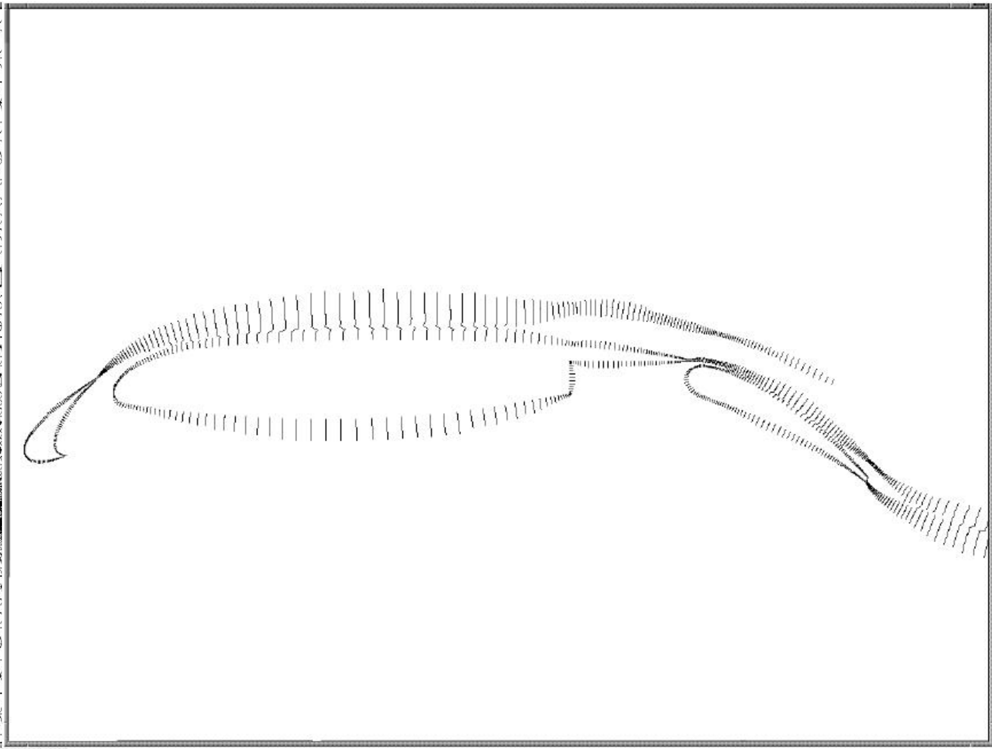


- Automated Graph-Based Coarsening Algorithm
- Coarse Levels are Graphs
- Coarse Level Operator by Galerkin Projection
- Grid independent convergence rates (order of magnitude improvement)

Enabling CFD Solver Developments

- Line solvers for Anisotropic problems
  - Lines constructed in mesh using weighted graph algorithm
  - Strong connections assigned large graph weight
  - (Block) Tridiagonal line solver similar to structured grids





Enabling CFD Solver Developments (1990 –2000)

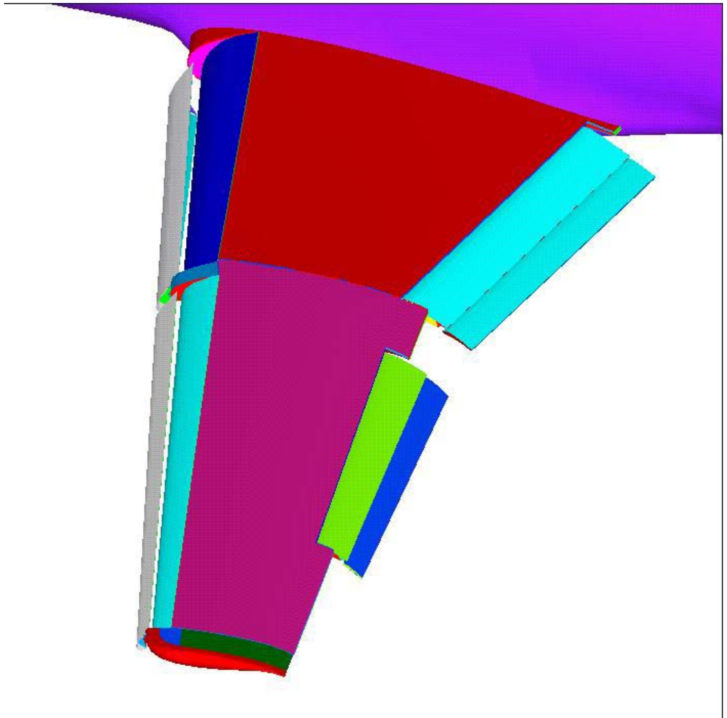
- Graph-based Partitioners for parallel load balancing
  - Metis, Chaco, Jostle
- Edge-data structure → graph of grid
- Agglomeration Multigrid levels = graphs
- Excellent load balancing up to 1000’s of processors
  - Homogeneous data-structures
  - (Versus multi-block / overlapping structured grids)

Practical Examples

- VGRIDns tetrahedral grid generator
- NSU3D Multigrid flow solver
  - Large scale massively parallel case
  - Fast turnaround medium size problem

NASA Langley Energy Efficient Transport

- Complex geometry
  - Wing-body, slat, double slotted flaps, cutouts
- Experimental data from Langley 14x22ft wind tunnel
  - Mach = 0.2, Reynolds=1.6 million
  - Range of incidences: -4 to 24 degrees

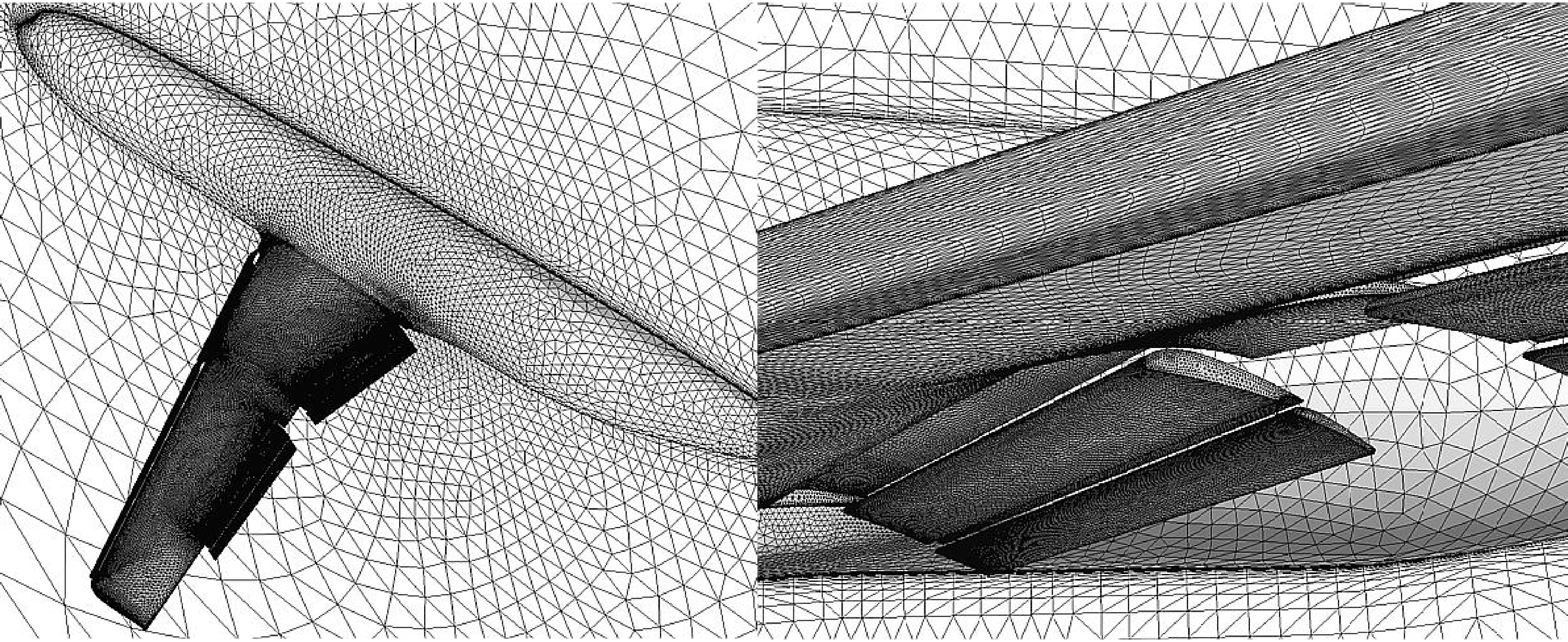


Initial Mesh Generation (VGRIDns)

S. Pirzadeh, NASA Langley

- Combined advancing layers- advancing front
  - Advancing layers: thin elements at walls
  - Advancing front: isotropic elements elsewhere
- Automatic switching from AL to AF based on:
  - Cell aspect ratio
  - Proximity of boundaries of other fronts
  - Variable height for advancing layers
- Background Cartesian grid for smooth spacing control
- Spanwise stretching
  - Factor of 3 reduction in grid size

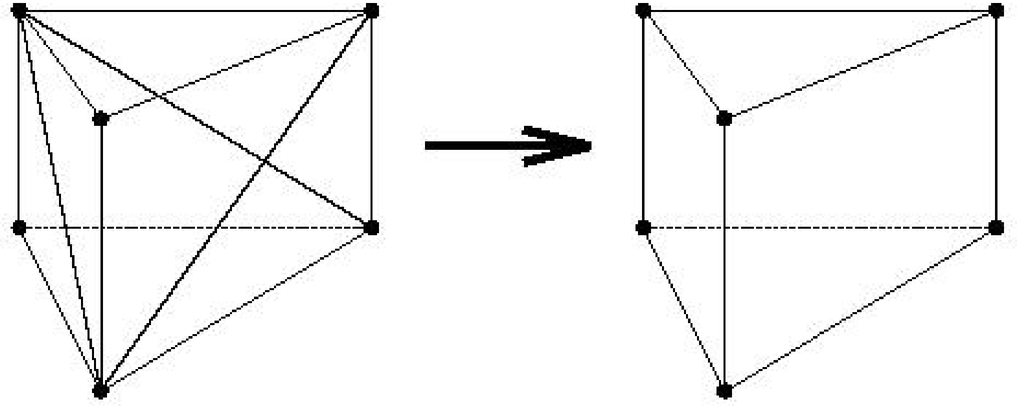
VGRID Tetrahedral Mesh



- 3.1 million vertices, 18.2 million tets, 115,489 surface pts
- Normal spacing: 1.35E-06 chords, growth factor=1.3

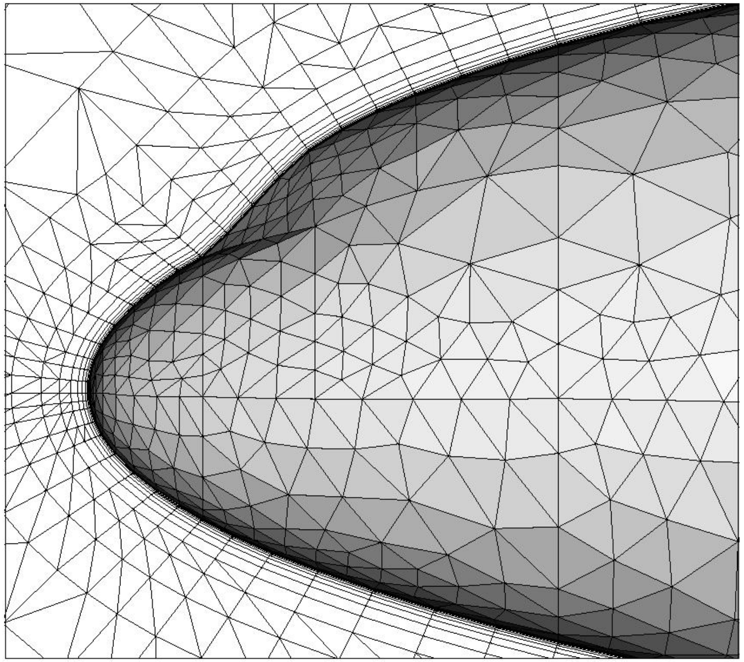
Prism Merging Operation

- Combine Tetrahedra triplets in advancing-layers region into prisms
  - Prisms entail lower complexity for solver
- VGRIDns identifies originating boundary point for ALR vertices
  - Used to identify candidate elements
  - Pyramids required as transitional elements



Prism Merging Operation

- Initial mesh: 18.2M Tetrahedra
- Merged mesh: 3.9M prisms, 6.6M Tets, 47K pyramids
  - 64% of Tetrahedra merged



Global Mesh Refinement

- High-resolution meshes require large parallel machines
- Parallel mesh generation difficult
  - Complicated logic
  - Access to commercial preprocessing, CAD tools
- Current approach
  - Generate coarse (O(10**6) vertices on workstation
  - Refine on supercomputer

Global Mesh Refinement

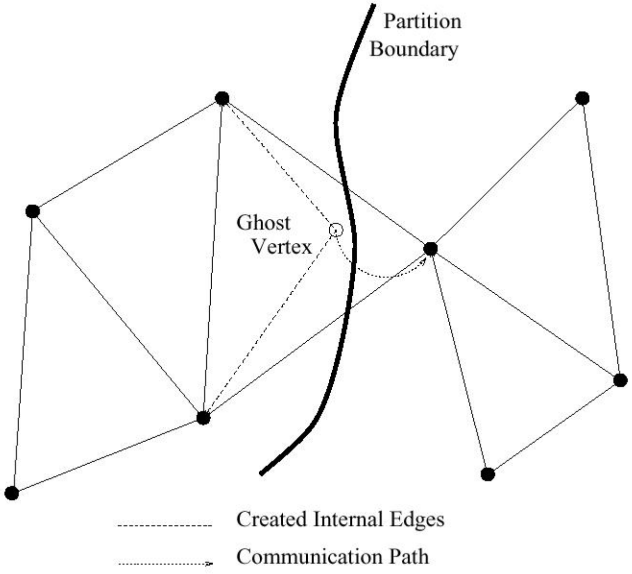
- Refinement achieved by element subdivision
- Global refinement: 8:1 increase in resolution
- In-Situ approach obviates large file transfers
- Initial mesh: 3.1 million vertices
  - 3.9M prisms, 6.6M Tets, 47K pyramids
- Refined mesh: 24.7 million vertices
  - 31M prisms, 53M Tets, 281K pyramids
  - Refinement operation: 10 Gbytes, 30 minutes sequentially

NSU3D Unstructured Mesh Navier-Stokes Solver

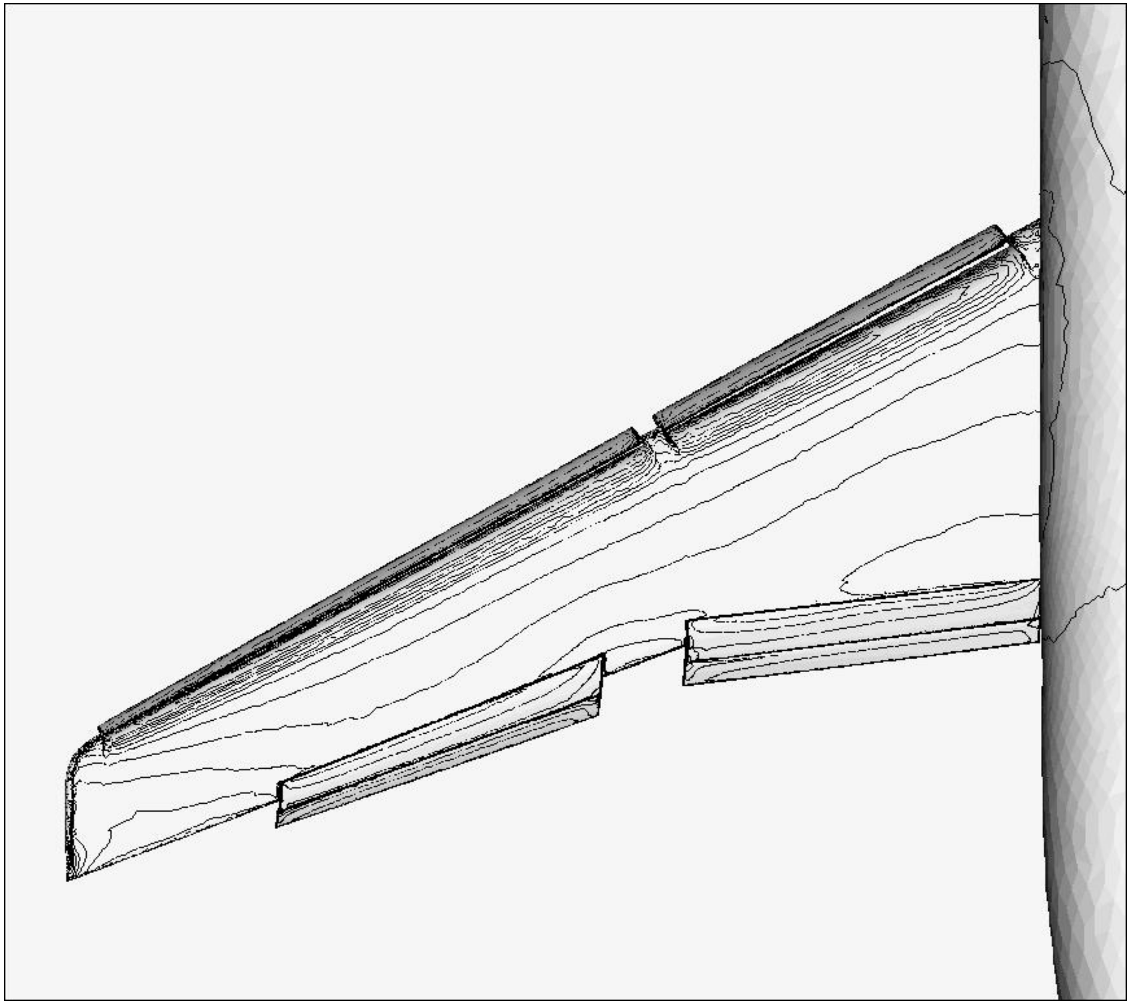
- Mixed element grids
  - Tetrahedra, prisms, pyramids, hexahedra
- Edge data-structure
- Line solver in BL regions near walls
- Agglomeration Multigrid acceleration
- Newton Krylov (GMRES) acceleration option
- Spalart-Allmaras 1 equation turbulence model

Parallel Implementation

- Domain decomposition with OpenMP/MPI communication
  - OpenMP on shared memory architectures
  - MPI on distributed memory architectures
  - Hybrid capability for clusters of SMPs
- Weighted graph partitioning (Metis)
- Coarse and fine MG levels partitioned independently

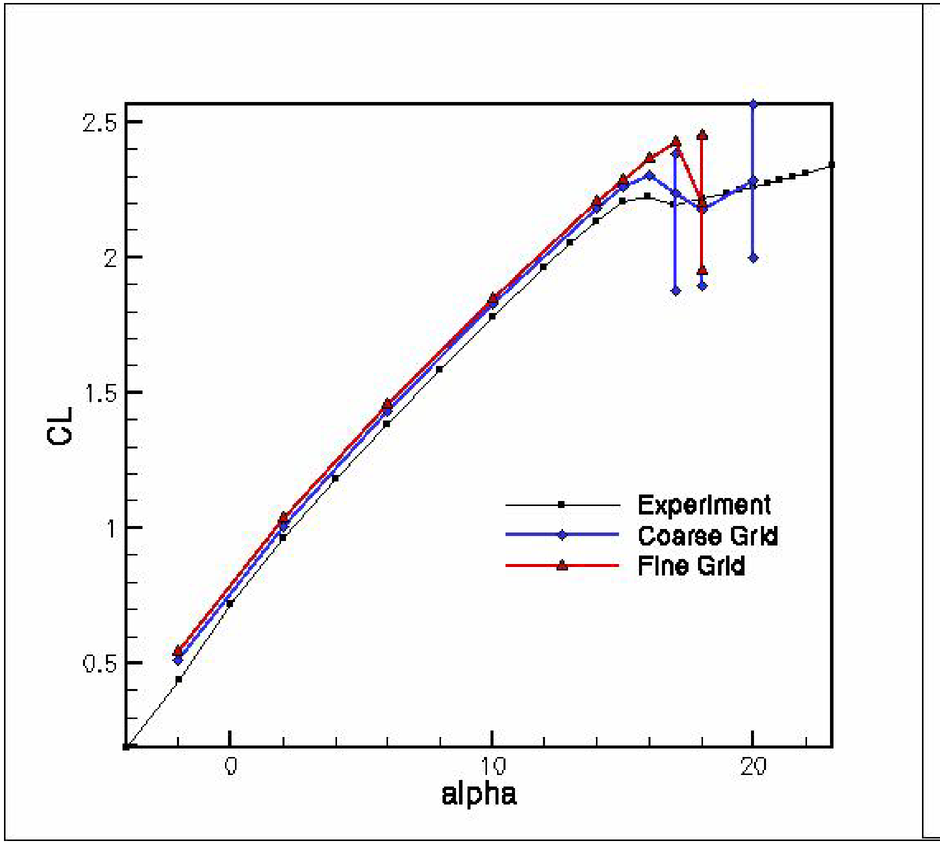


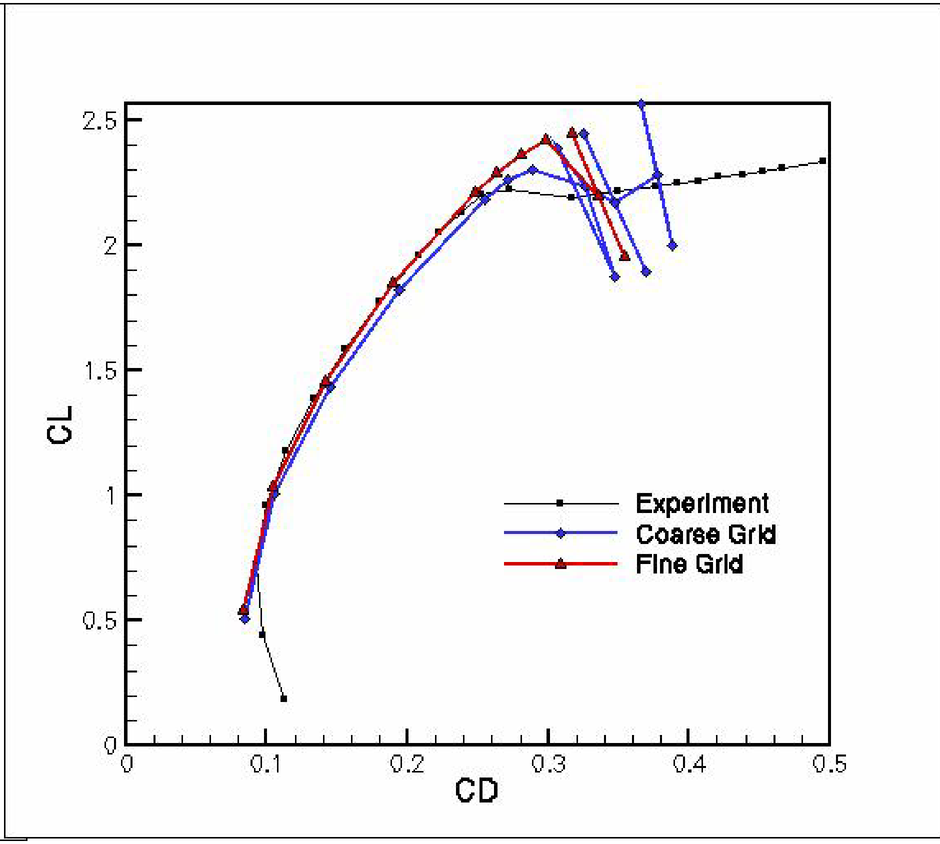
Computed Pressure Contours on Coarse Grid



- Mach=0.2, Incidence=10 degrees, Re=1.6M

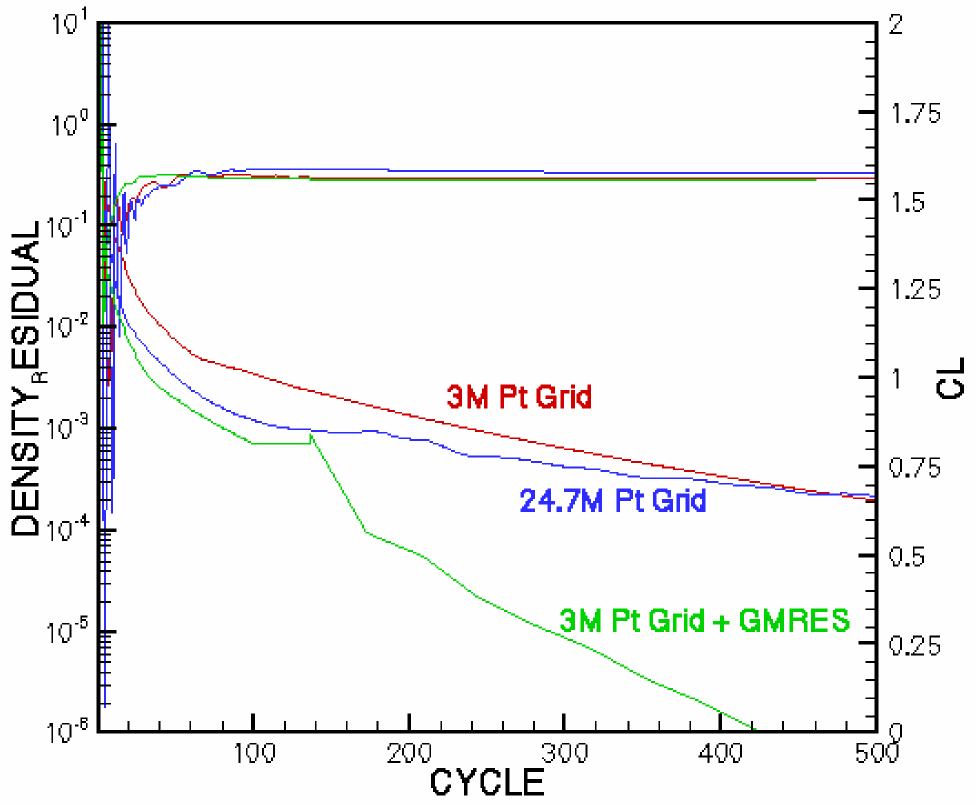
Computed Versus Experimental Results





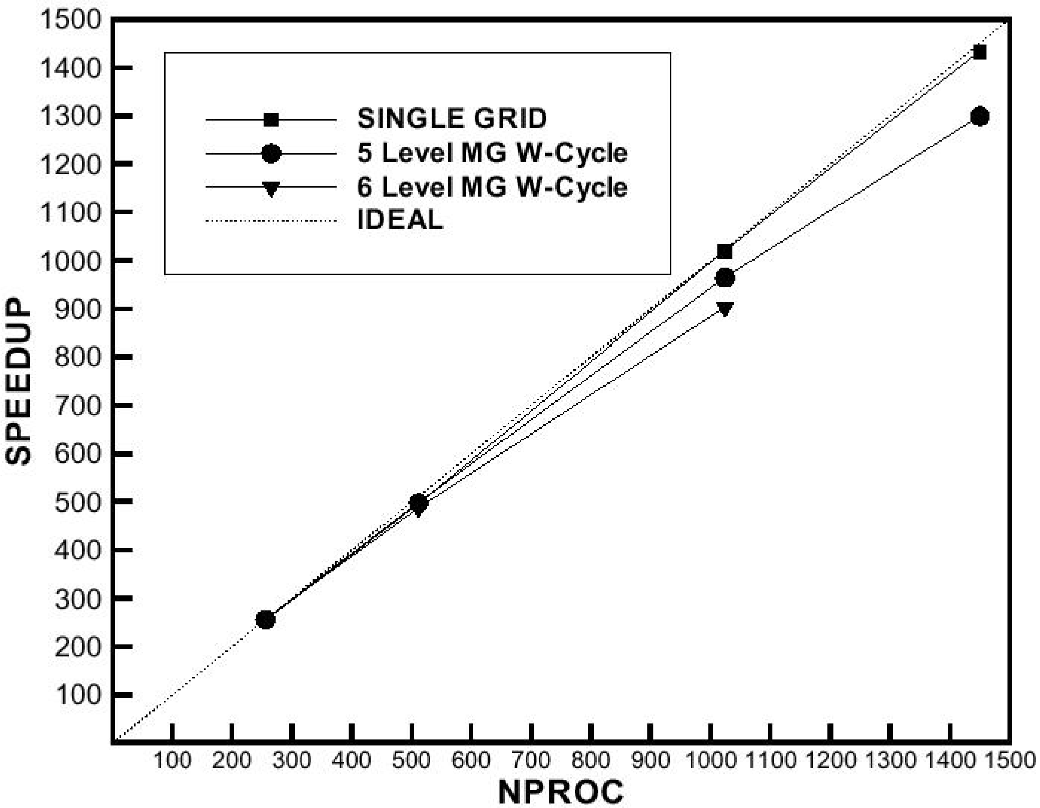
- Good drag prediction
- Discrepancies near stall

Multigrid Convergence History



- Mesh independent property of Multigrid
- GMRES effective but requires extra memory

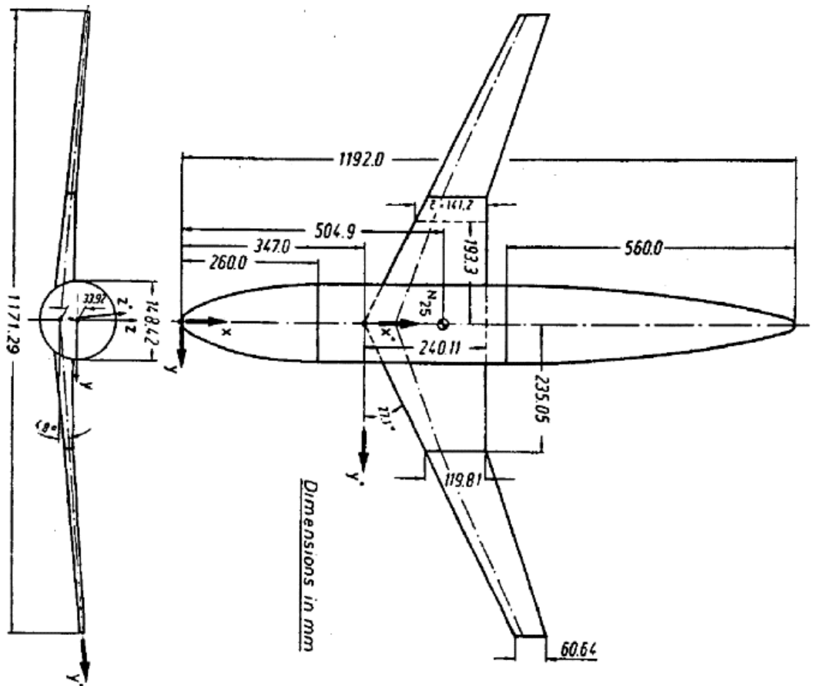
Parallel Scalability



24.7 Million Pt Case (5 Multigrid Levels)			
Platform	No. of Procs	Time/Cyc	Gflop/s
T3E-600	512	28.1	22.0
T3E-1200e	256	38.3	16.1
T3E-1200e	512	19.7	31.4
T3E-1200e	1024	10.1	61.0
T3E-1200e	1450	7.54	82.0

- Good overall Multigrid scalability
  - Increased communication due to coarse grid levels
  - Single grid solution impractical (>100 times slower)
- 1 hour soution time on 1450 PEs

AIAA Drag Prediction Workshop (2001)

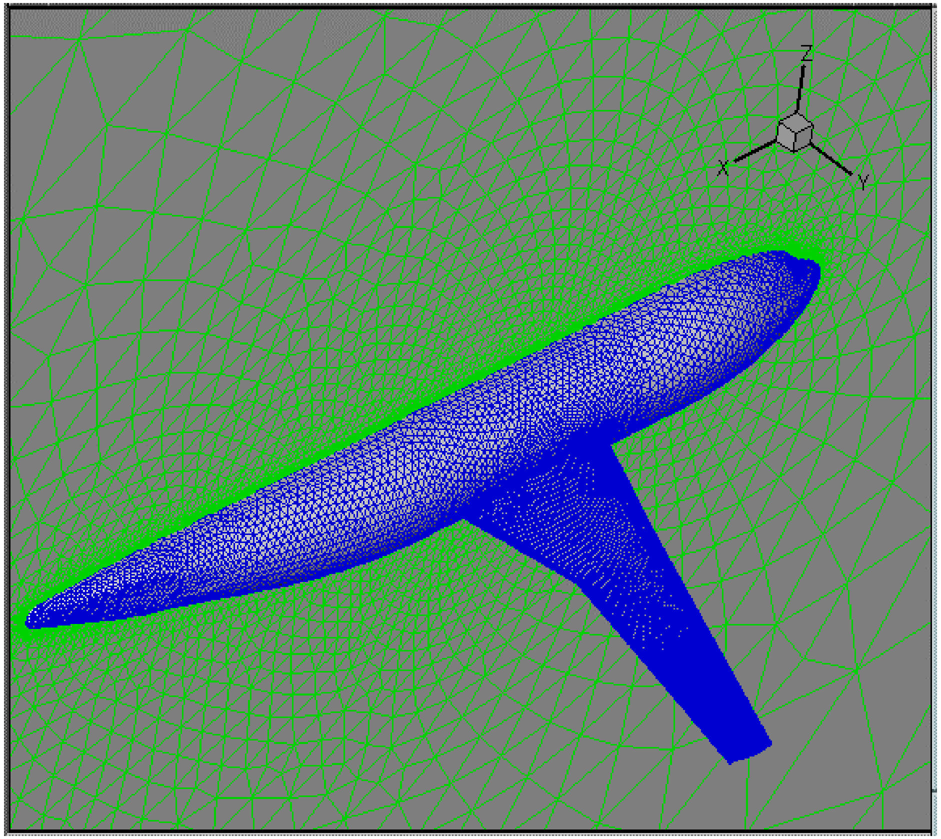


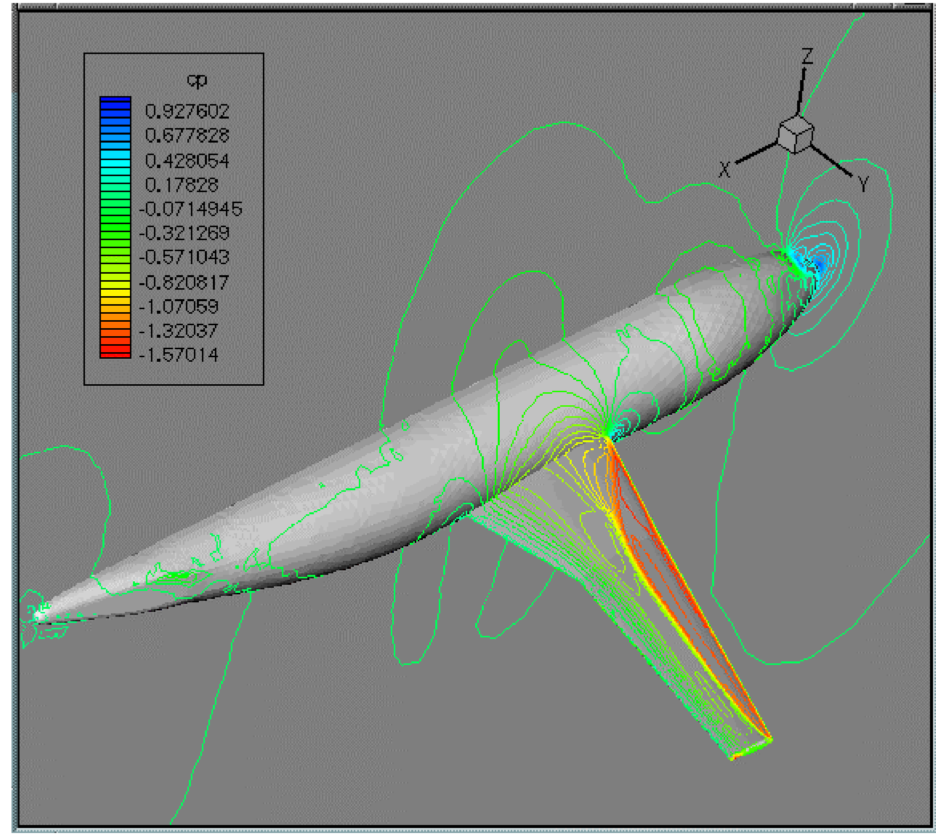
- Transonic wing-body configuration
- Typical cases required for design study
  - Matrix of mach and CL values
  - Grid resolution study
- Follow on with engine effects (2003)

Cases Run

- Baseline grid: 1.6 million points
  - Full drag polars for Mach=0.5,0.6,0.7,0.75,0.76,0.77,0.78,0.8
  - Total = 72 cases
- Medium grid: 3 million points
  - Full drag polar for each mach number
  - Total = 48 cases
- Fine grid: 13 million points
  - Drag polar at mach=0.75
  - Total = 7 cases

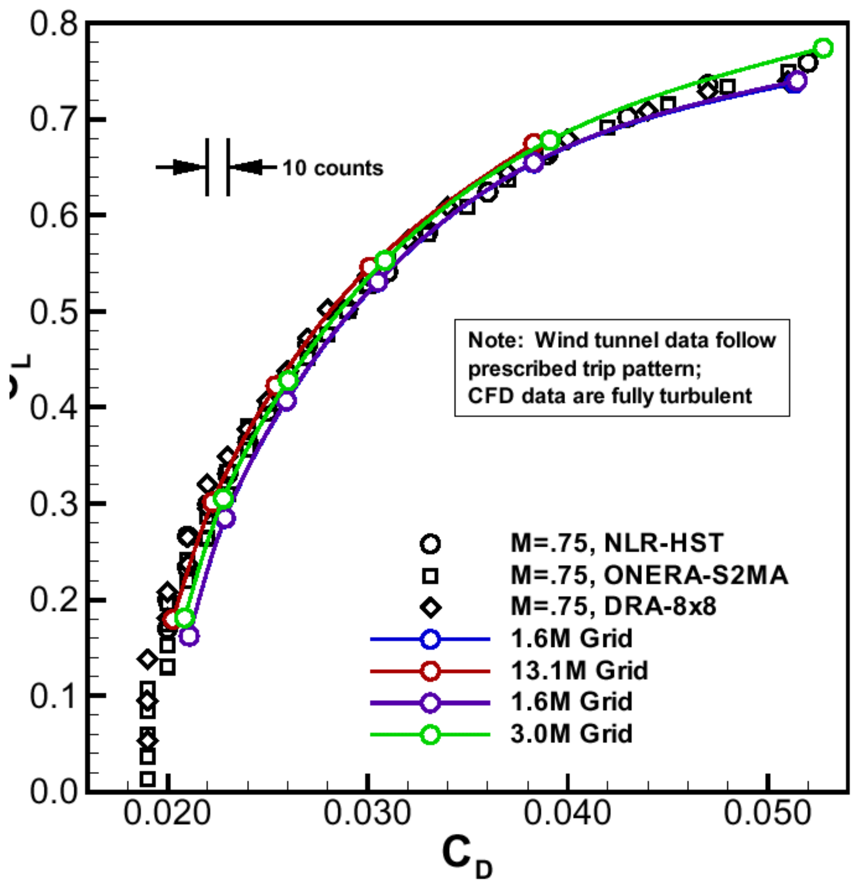
Sample Solution (1.65M Pts)





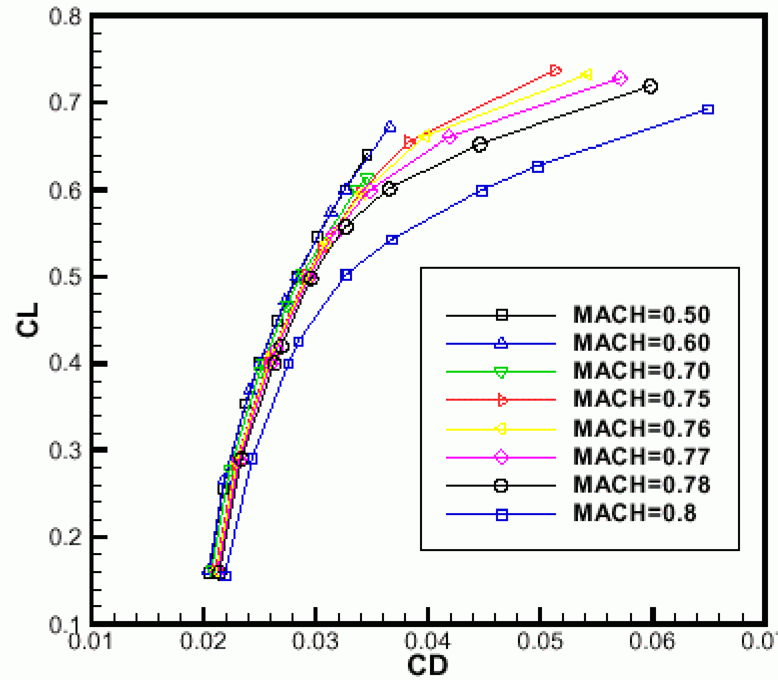
- Mach=0.75, CL=0.6, Re=3M
- 2.5 hours on 16 Pentium IV 1.7GHz

Drag Polar at Mach = 0.75



- Grid resolution study
- Good comparison with experimental data

Cases Run on ICASE Cluster



- 120 Cases (excluding finest grid)
- About 1 week to compute all cases

Current and Future Issues

- Adaptive mesh refinement
- Moving geometry and mesh motion
- Moving geometry and overlapping meshes
- Requirements for gradient-based design
- Implications for higher-order Discretizations

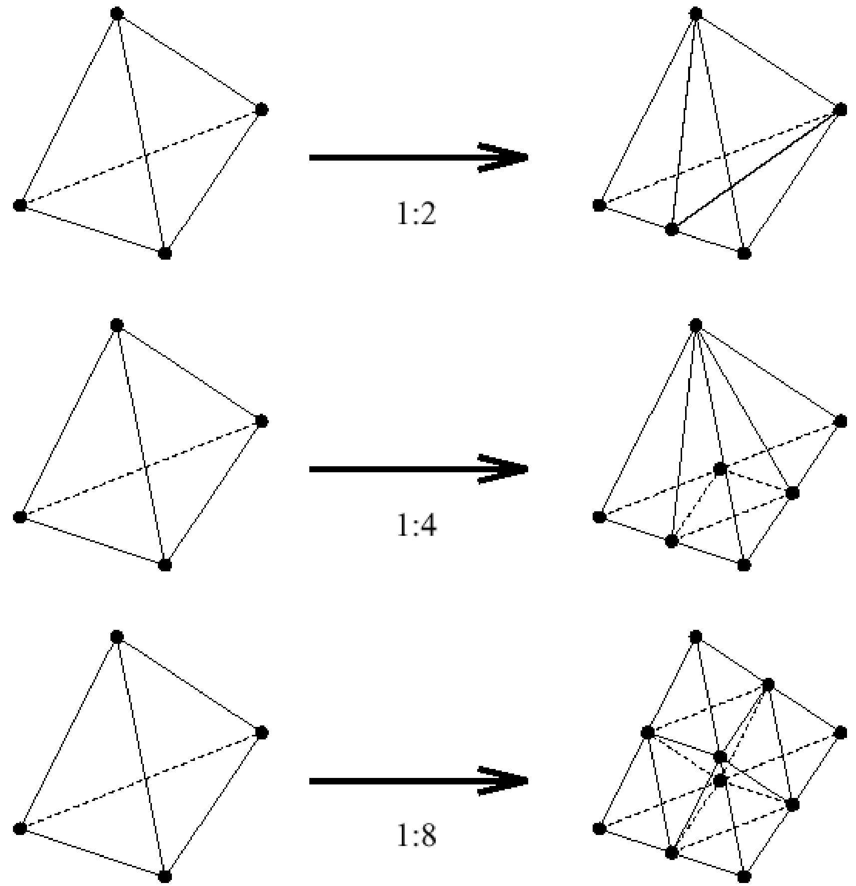
Adaptive Meshing

- Potential for large savings through optimized mesh resolution
  - Well suited for problems with large range of scales
  - Possibility of error estimation / control
  - Requires tight CAD coupling (surface pts)
- Mechanics of mesh adaptation
- Refinement criteria and error estimation

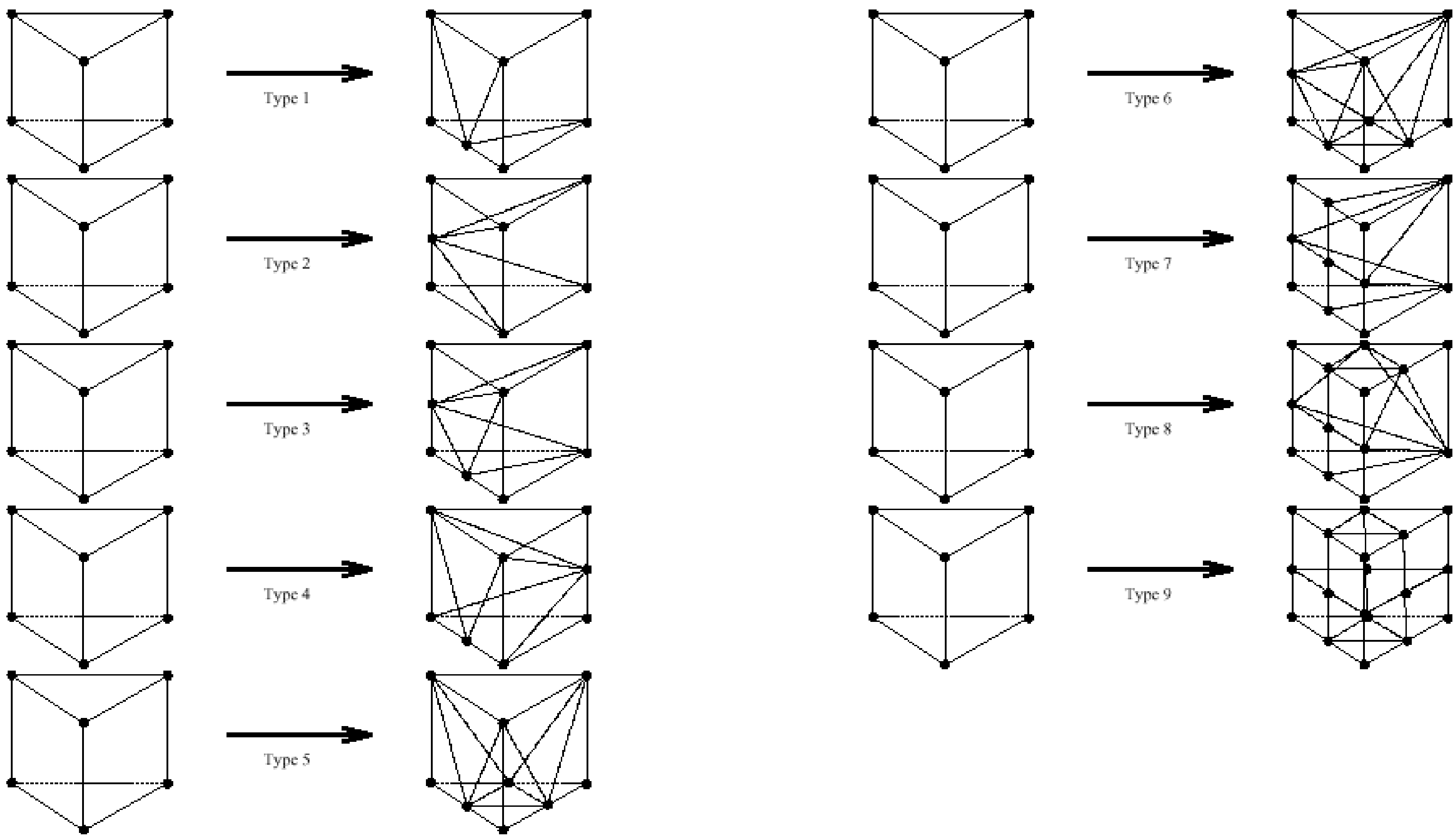
Mechanics of Adaptive Meshing

- Various well know isotropic mesh methods
  - Mesh movement
    - Spring analogy
    - Linear elasticity
  - Local Remeshing
  - Delaunay point insertion/Retriangulation
  - Edge-face swapping
  - Element subdivision
    - Mixed elements (non-simplicial)
    - Anisotropic subdivision required in transition regions

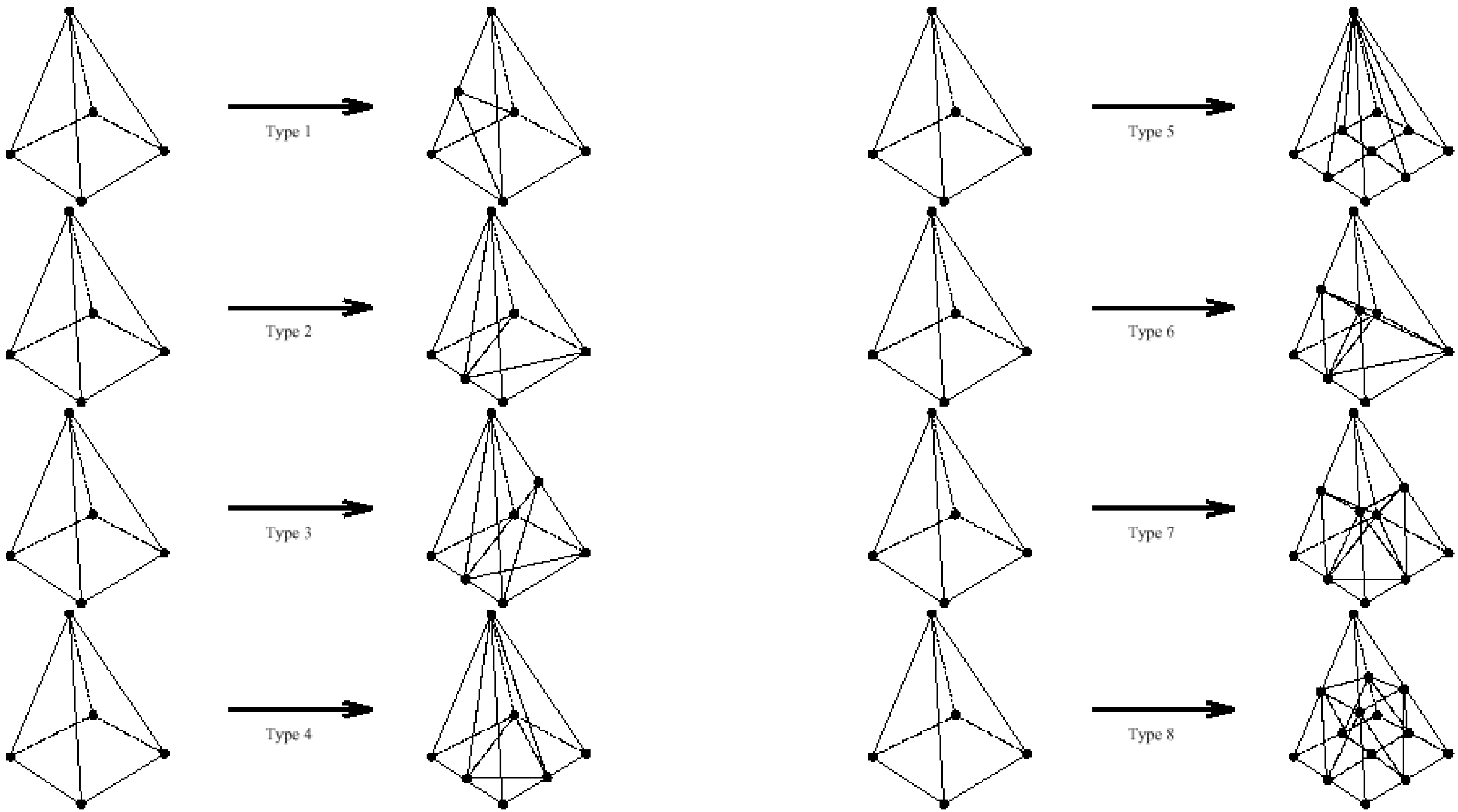
Subdivision Types for Tetrahedra



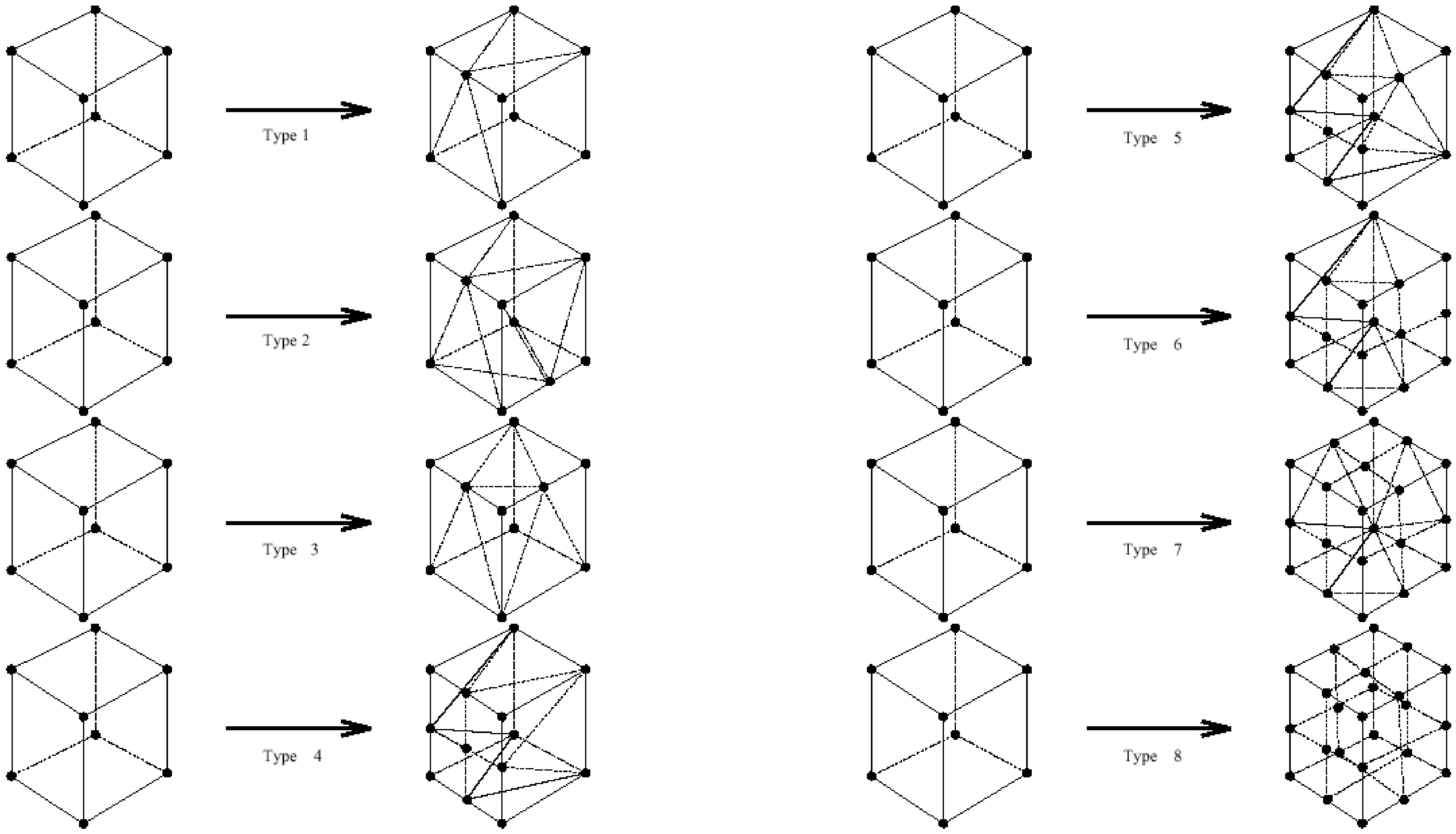
Subdivision Types for Prisms



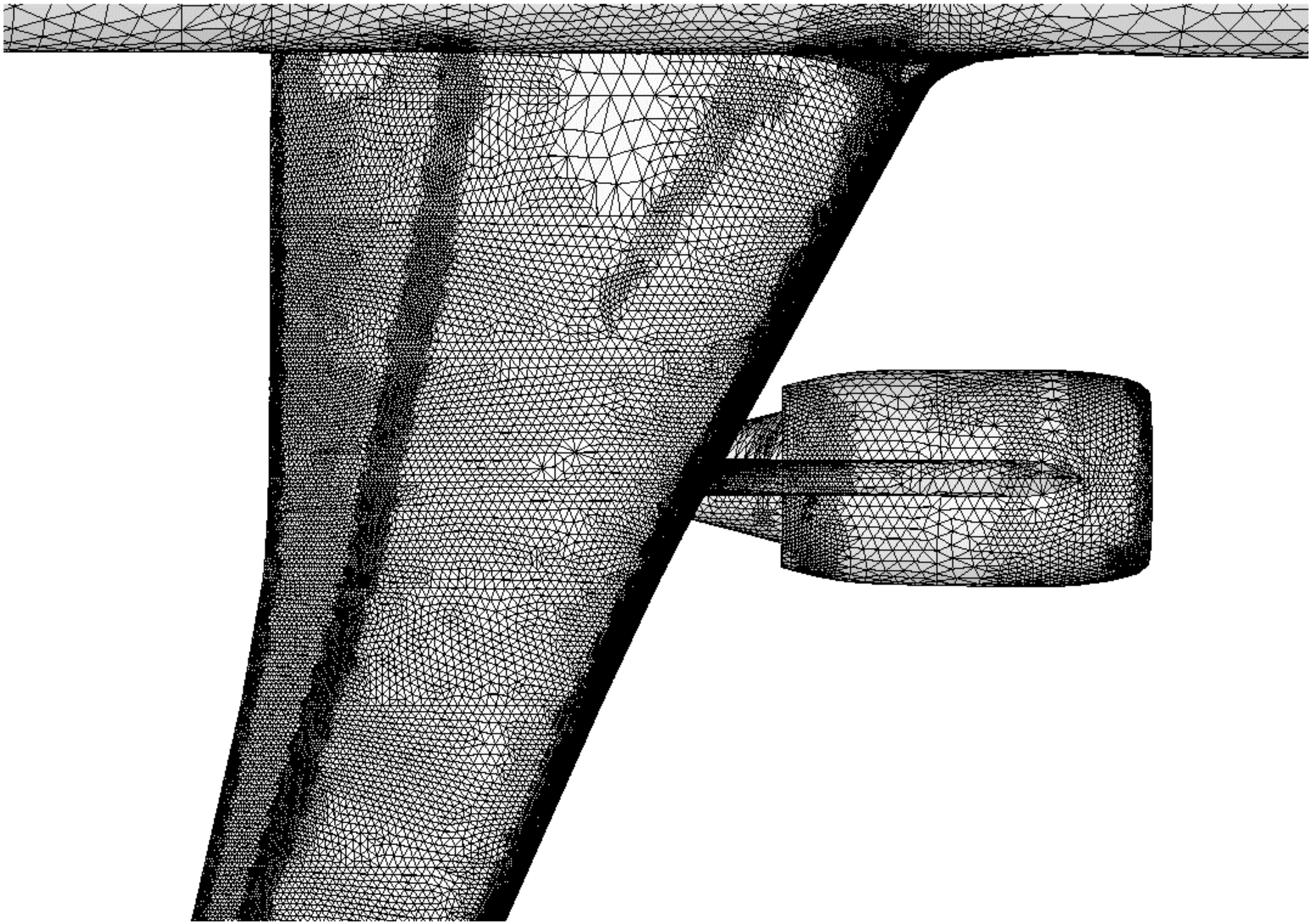
Subdivision Types for Pyramids



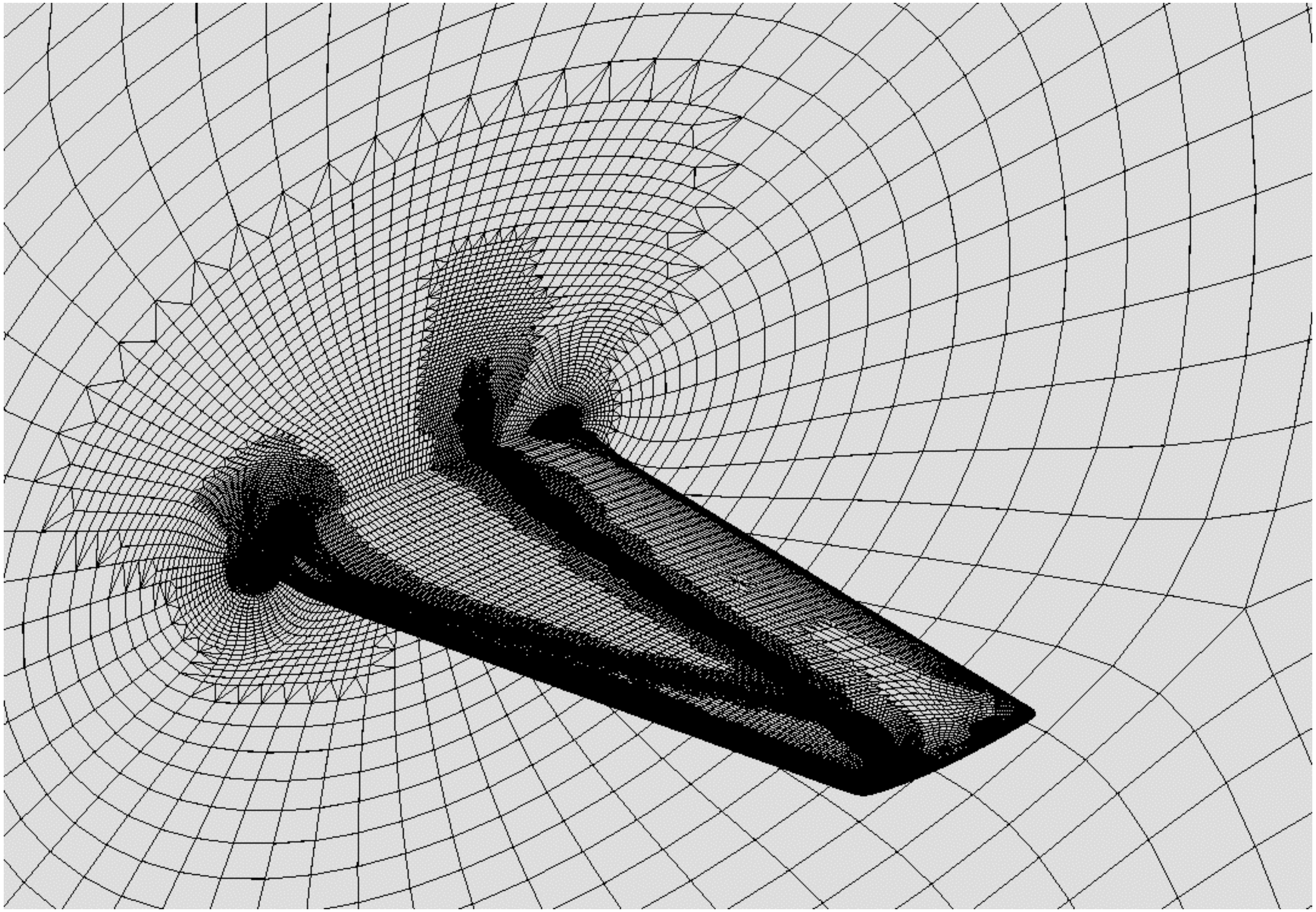
Subdivision Types for Hexahedra



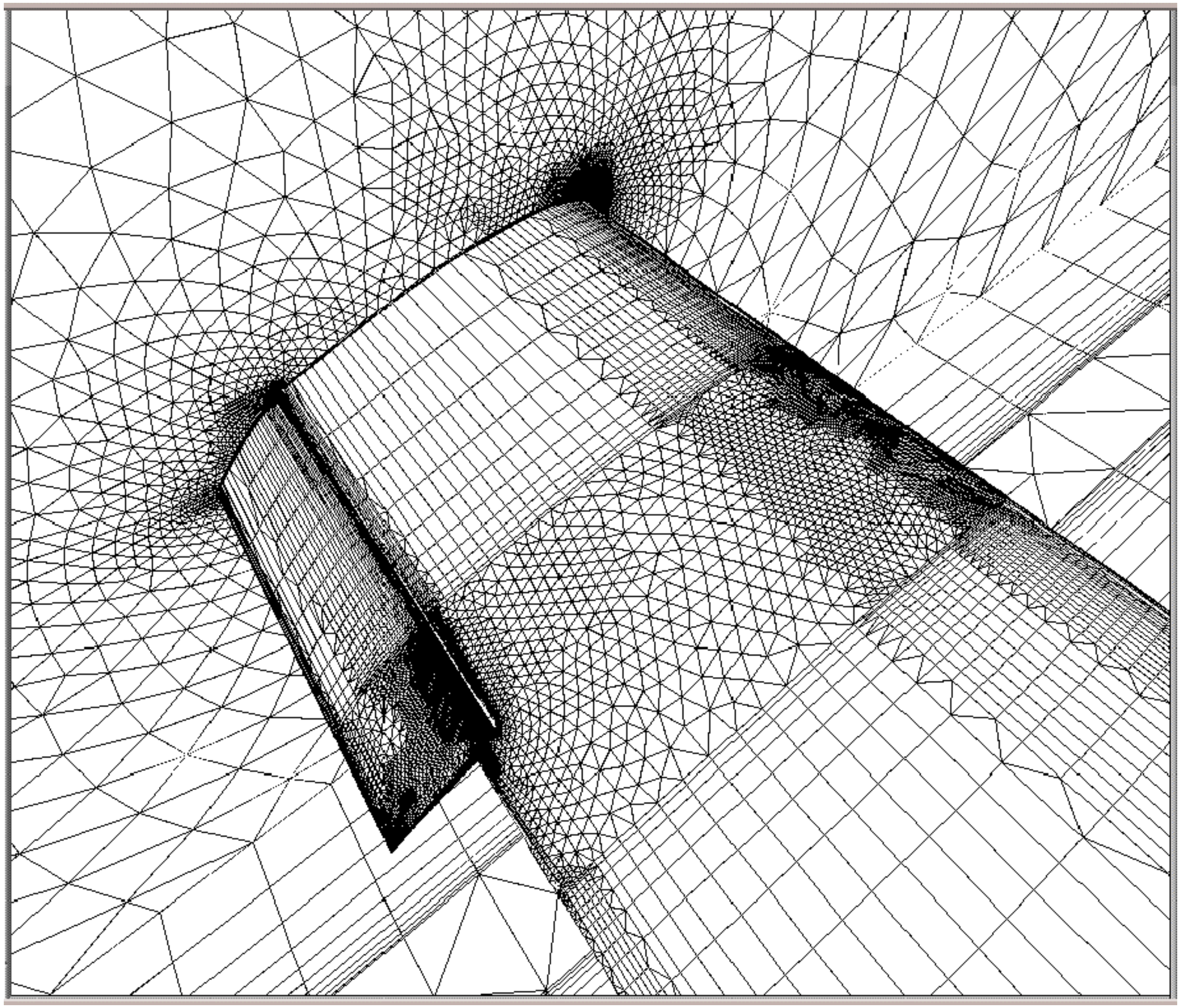
Adaptive Tetrahedral Mesh by Subdivision



Adaptive Hexahedral Mesh by Subdivision



Adaptive Hybrid Mesh by Subdivision



Anisotropic Adaptation Methods

- Large potential savings for 1 or 2D features
  - Directional subdivision
    - Assumes element faces to line up with flow features
    - Combine with mesh motion
  - Mapping techniques
    - Hessian based
    - Grid quality

Refinement Criteria

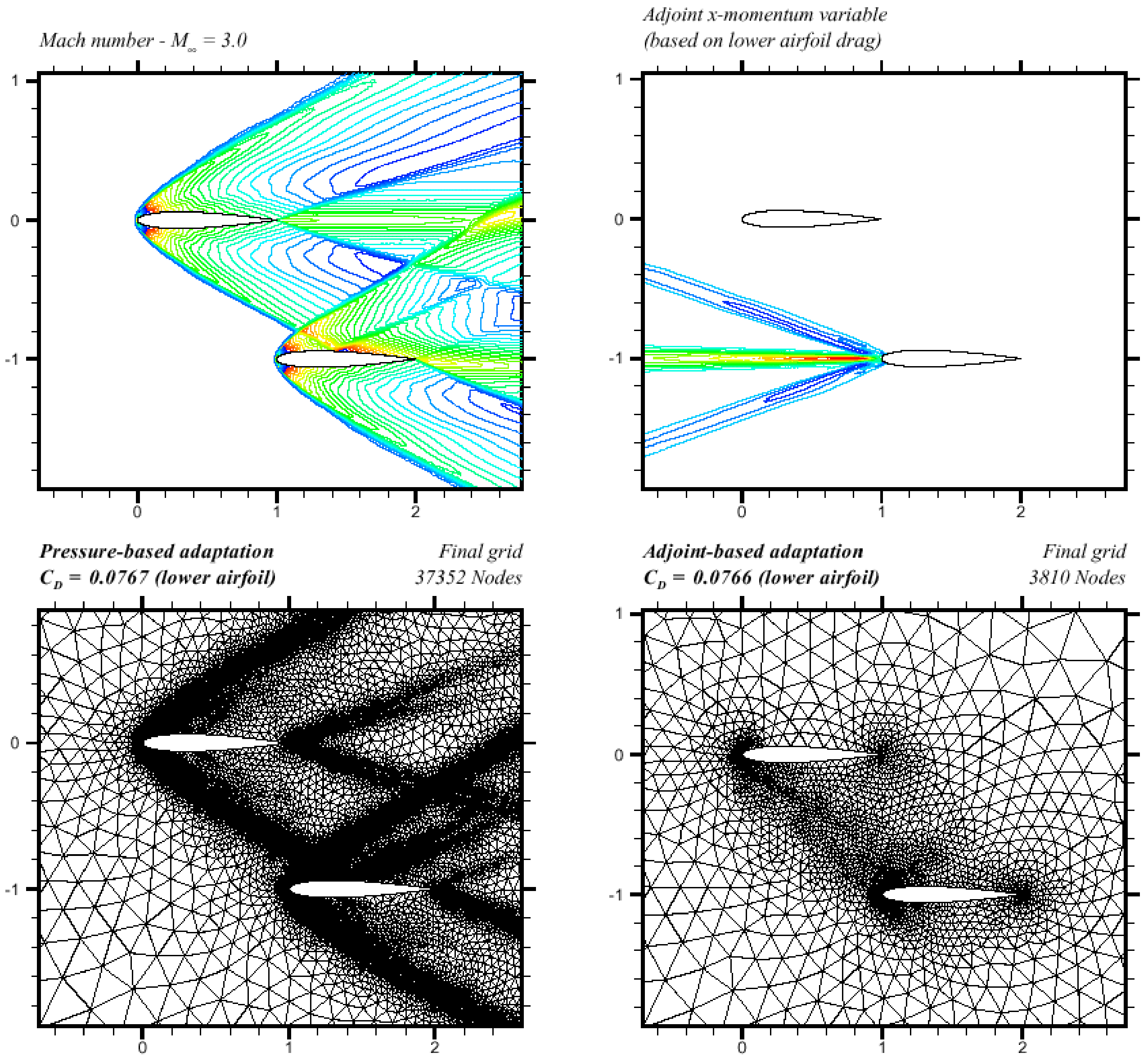
- Weakest link of adaptive meshing methods
  - Obvious for strong features
  - Difficult for non-local (ie. Convective) features
    - eg. Wakes
  - Analysis assumes in asymptotic error convergence region
    - Gradient based criteria
    - Empirical criteria
- Effect of variable discretization error in design studies, parameter sweeps

Adjoint-based Error Prediction

- Compute sensitivity of global cost function to local spatial grid resolution
- Key on important output, ignore other features
  - Error in engineering output, not discretization error
    - e.g. Lift, drag, or sonic boom …
- Captures non-local behavior of error
  - Global effect of local resolution
- Requires solution of adjoint equations
  - Adjoint techniques used for design optimization

Adjoint-based Mesh Adaptation Criteria

Reproduced from Venditti and Darmofal (MIT, 2002)



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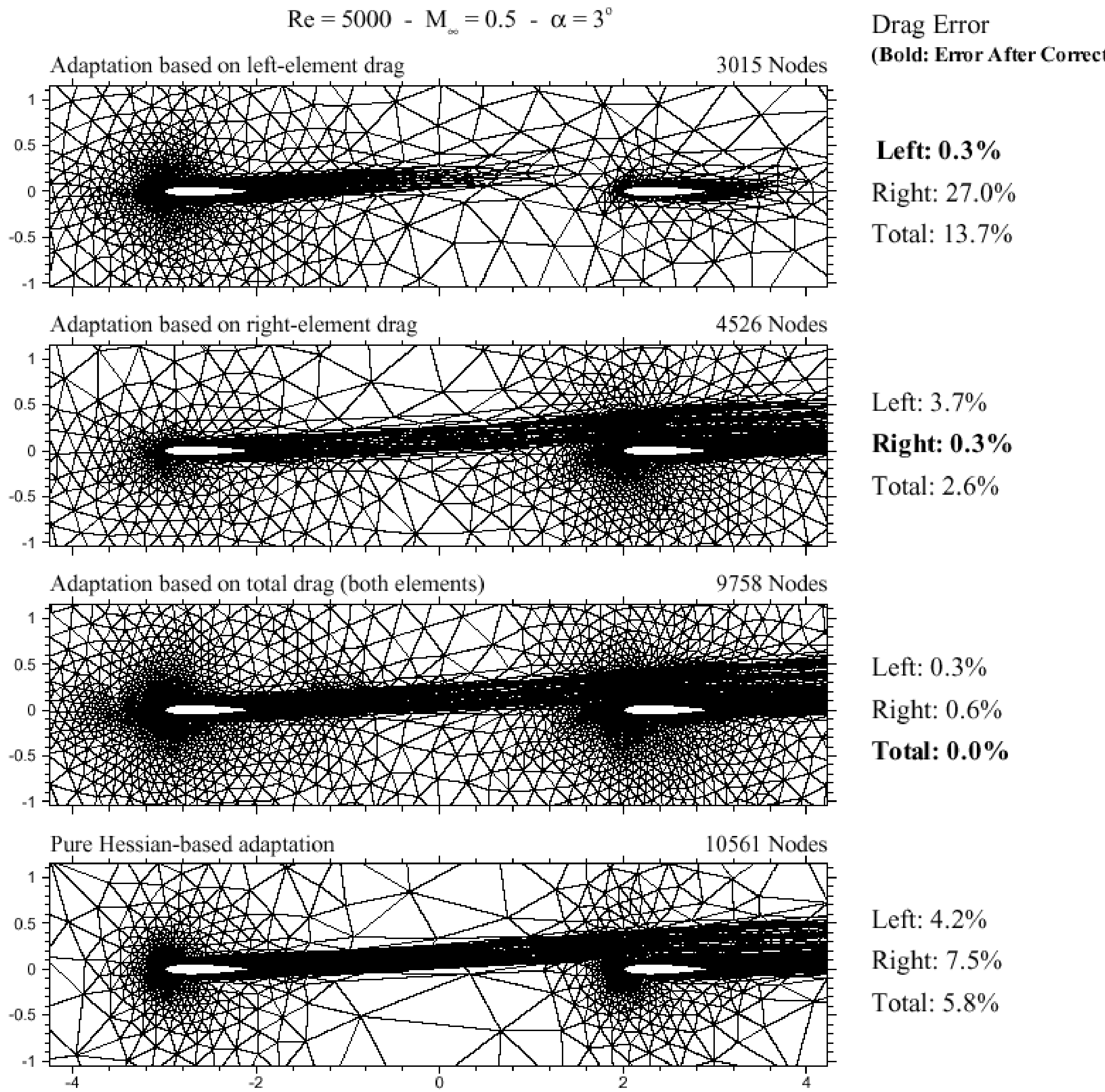

Adjoint-based Mesh Adaptation Criteria

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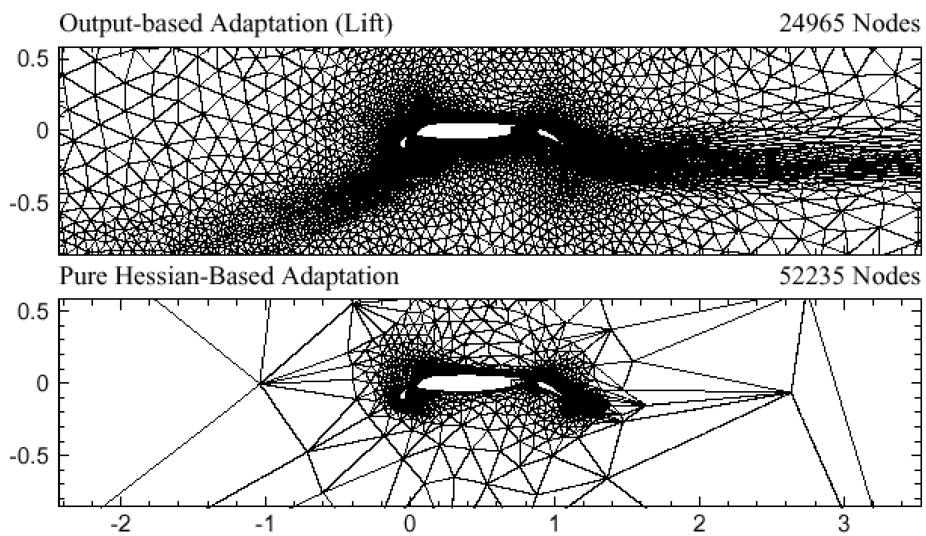
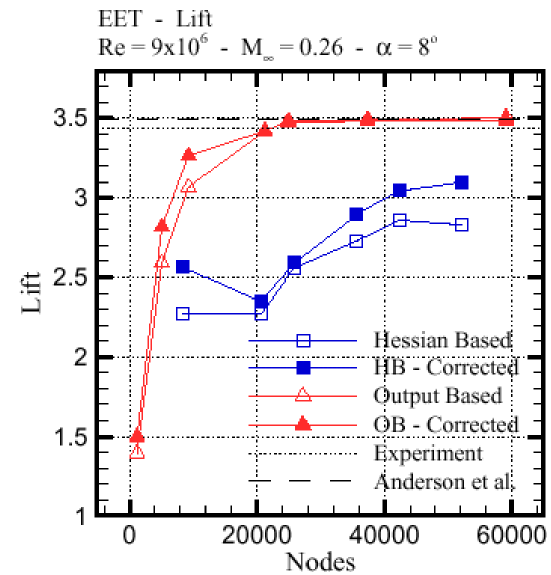
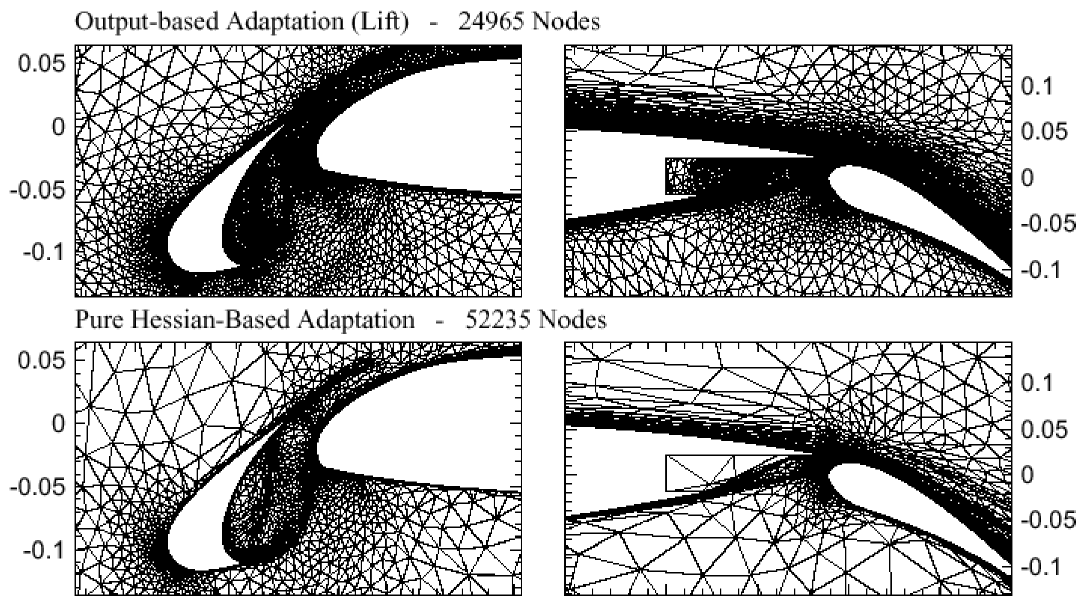


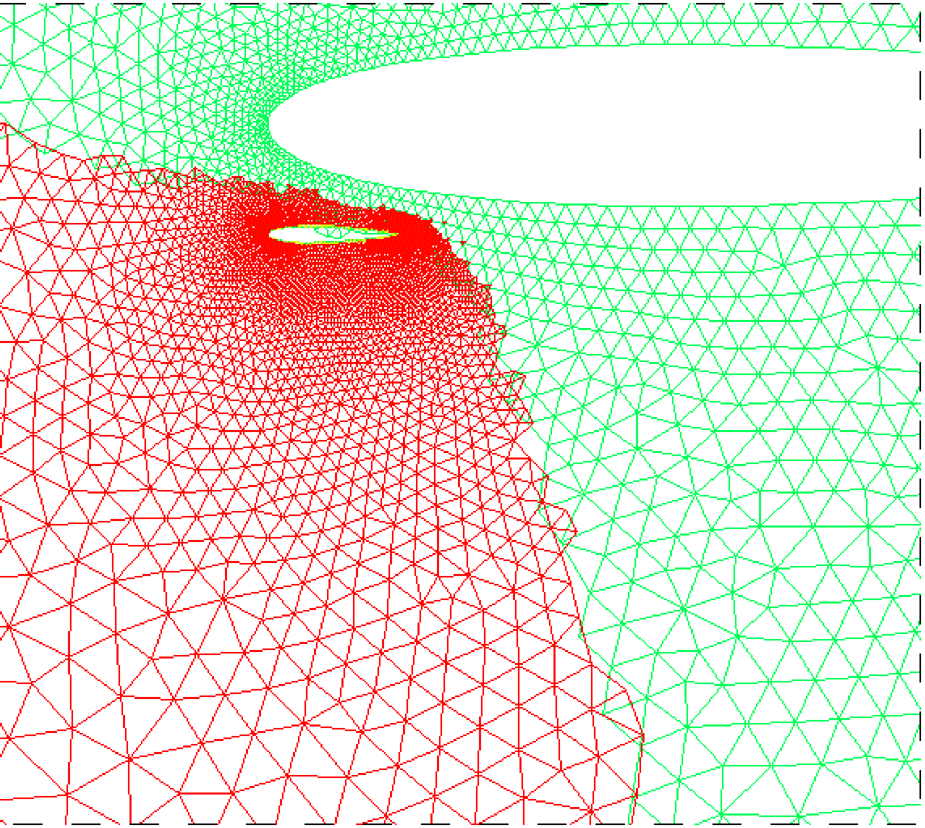


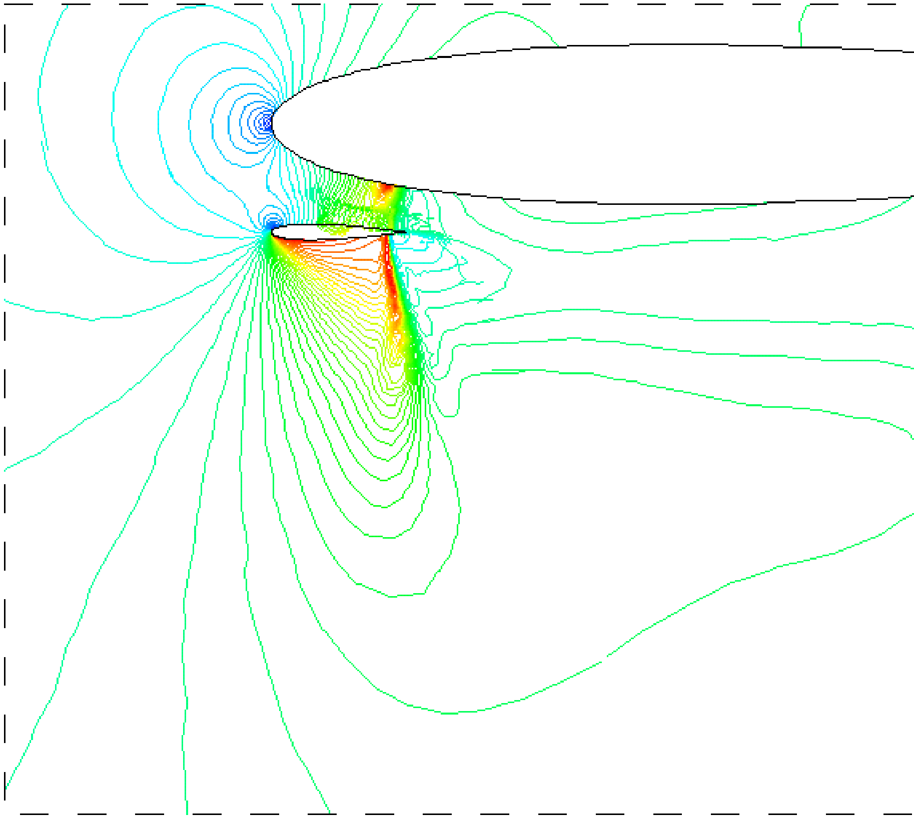


Overlapping Unstructured Meshes

- Alternative to moving mesh for large scale relative geometry motion
- Multiple overlapping meshes treated as single data-structure
  - Dynamic determination of active/inactive/ghost cells
- Advantages for parallel computing
  - Obviates dynamic load rebalancing required with mesh motion techniques
  - Intergrid communication must be dynamically recomputed and rebalanced
    - Concept of Rendez-vous grid (Plimpton and Hendrickson)

Overlapping Unstructured Meshes





- Simple 2D transient example

Gradient-based Design Optimization

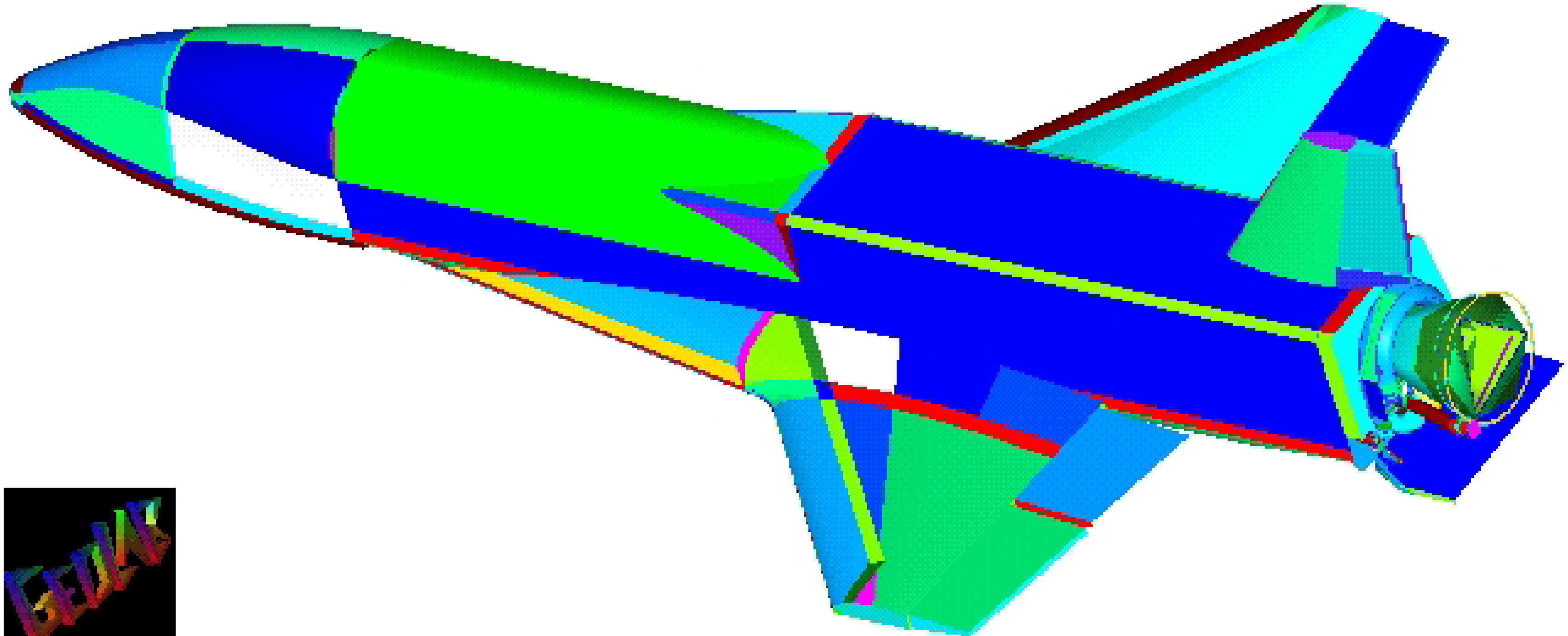
- Minimize Cost Function F with respect to design variables v, subject to constraint $R(w) = 0$
  - F = drag, weight, cost
  - v = shape parameters
  - w = Flow variables
  - $R(w) = 0$ → Governing Flow Equations
- Gradient Based Methods approach minimum along direction : $-\frac{\partial F}{\partial v}$

Grid Related Issues for Gradient-based Design

- Parametrization of CAD surfaces
- Consistency across disciplines
  - eg. CFD, structures,…
- Surface grid motion
- Interior grid motion
- Grid sensitivities
- Automation / Parallelization

Preliminary Design Geometry

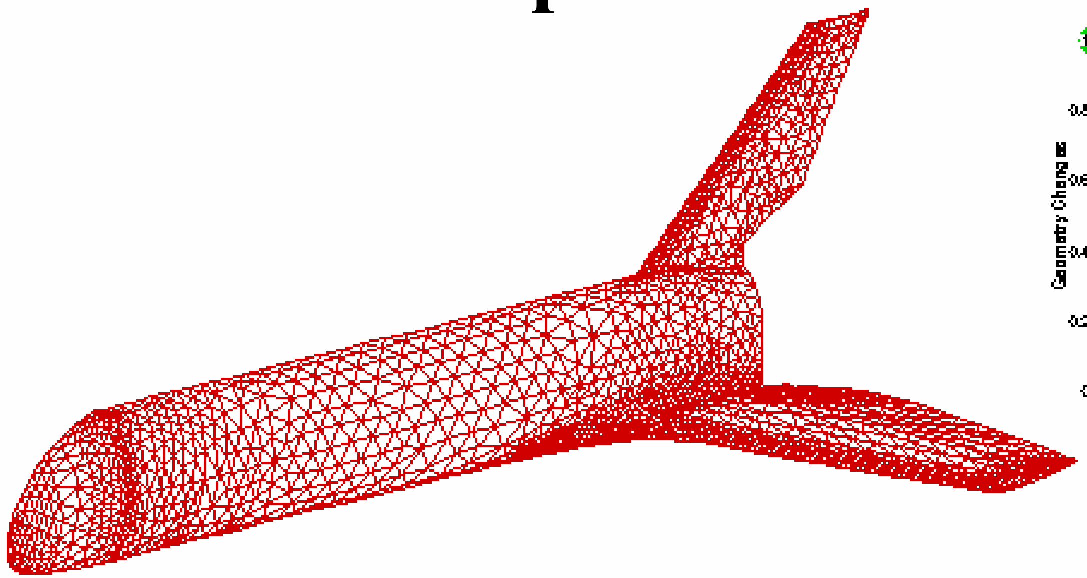
X34 CAD Model

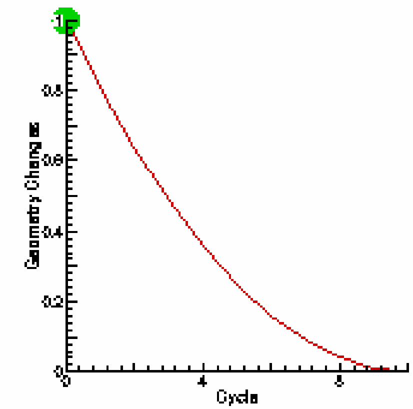


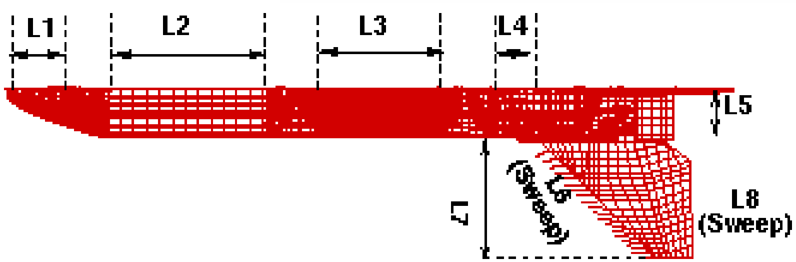
23,555 curves and surfaces

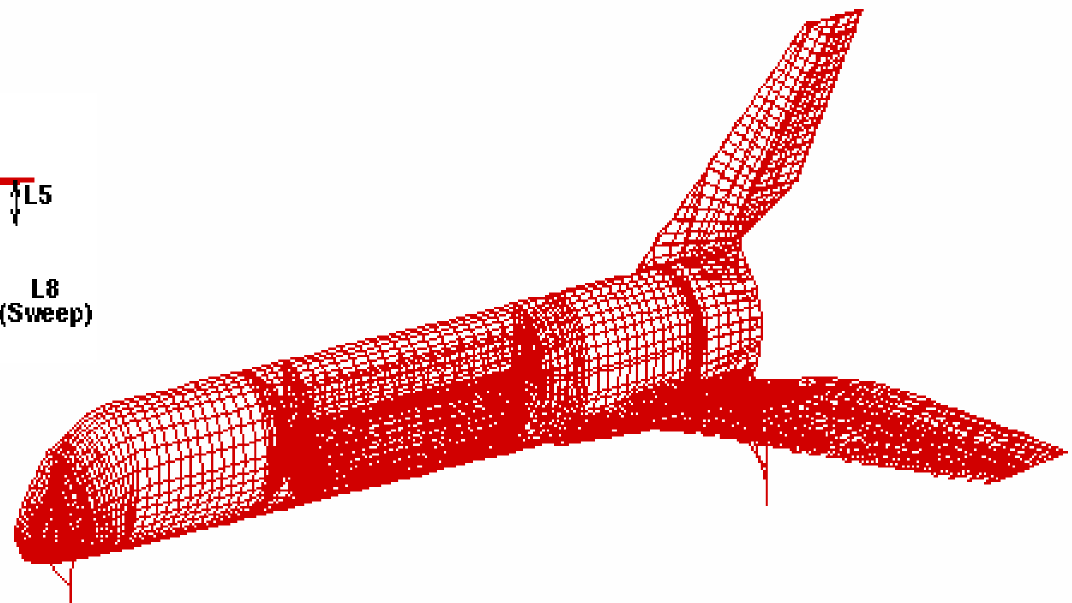
c/o J. Samareh, NASA Langley

Launch Vehicle Shape Parameterization









c/o J. Samareh, NASA Langley

Sensitivity Analysis

objective function

(e.g., Stress, C_D)

v design variables

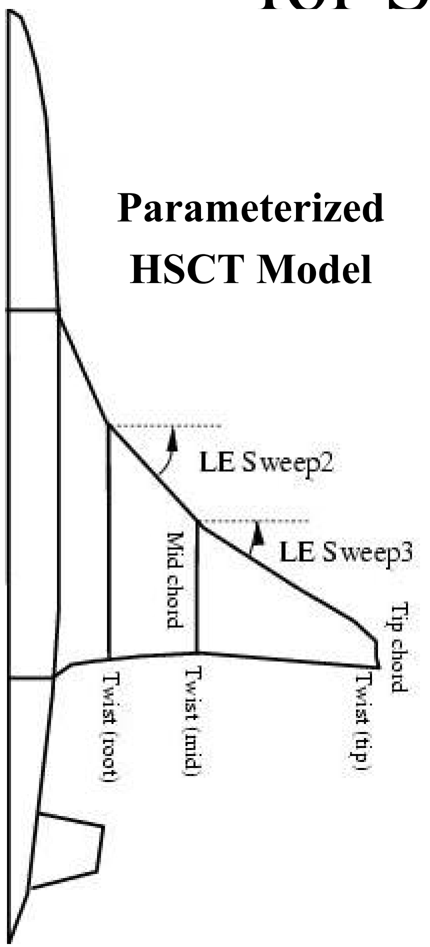
(e.g., span, camber)

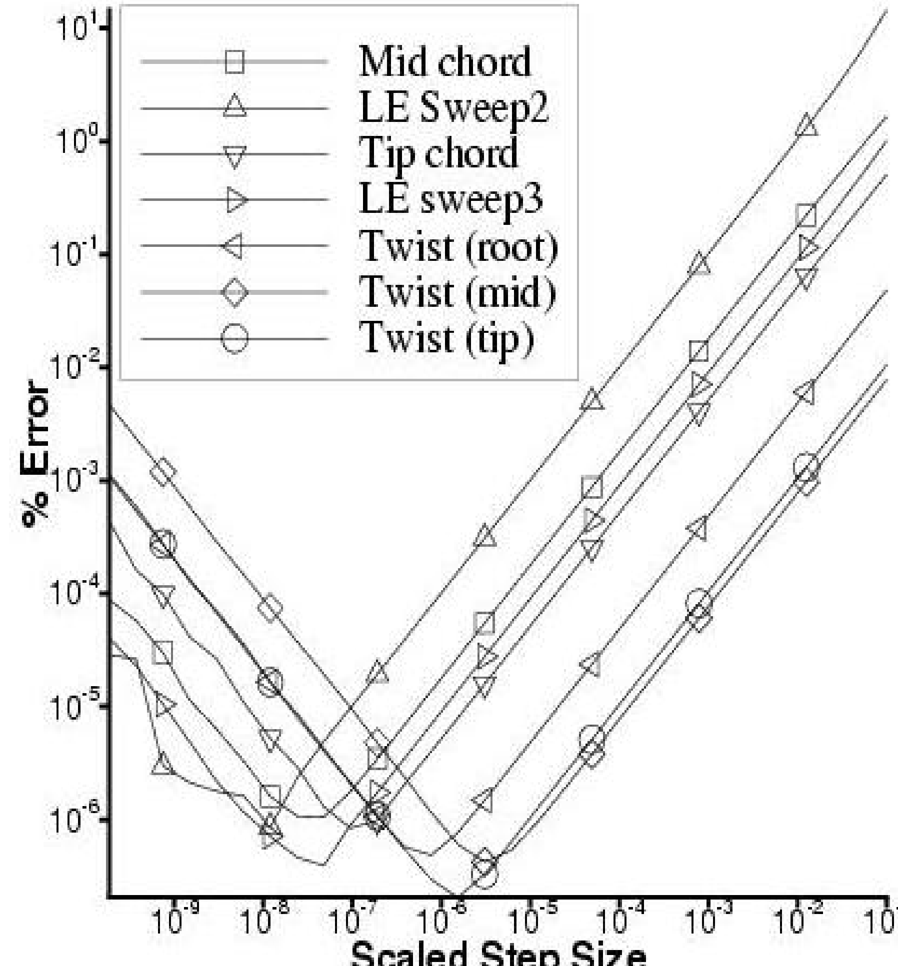
$$\frac{\partial F}{\partial \overline{v}} = \underbrace{\frac{\partial F}{\partial Grid_f}}_{\text{analysis code}} \; x \; \underbrace{\frac{\partial Grid_f}{\partial Grid_s}}_{\text{field grid generator}} \; x \; \underbrace{\frac{\partial Grid_s}{\partial Geometry}}_{\text{surface grid generator}} \; x \; \underbrace{\frac{\partial Geometry}{\partial \overline{v}}}_{\text{geometry modeler (CAD)}}$$

- Manual differentiation
- Automatic differentiation tools (e.g., ADIFOR and ADIC)
- Complex variables
- Finite-difference approximations

c/o J. Samareh, NASA Langley

Finite-Difference Approximation Error for Sensitivity Derivatives





c/o J. Samareh, NASA Langley

Grid Sensitivities

$$\frac{\partial \mathrm{Grid}_f}{\partial \overline{\mathrm{v}}} = \frac{\partial \mathrm{Grid}_f}{\partial \mathrm{Grid}_s} x \frac{\partial \mathrm{Grid}_s}{\partial \mathrm{Geometry}} x \frac{\partial \mathrm{Geometry}}{\partial \overline{\mathrm{v}}}$$

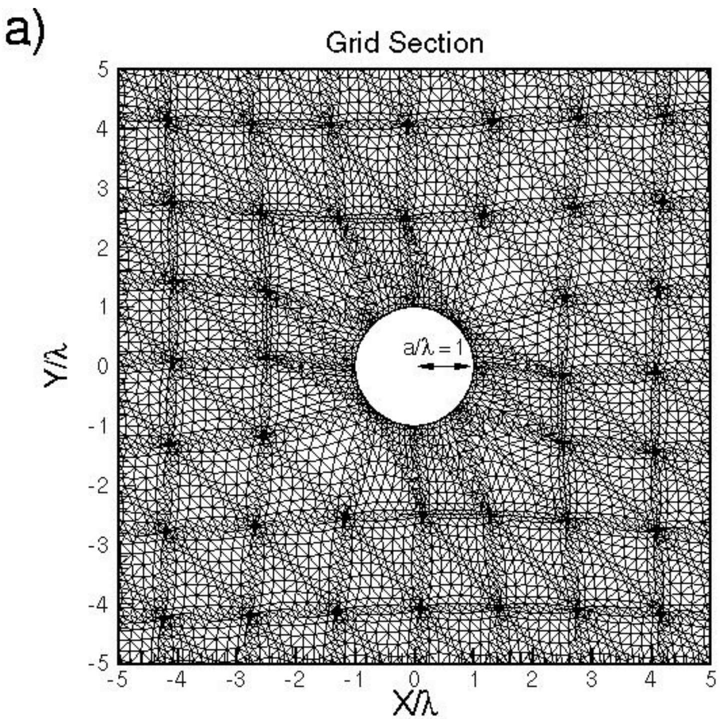
- Ideally should be available from grid/cad software
  - Analytical formulation most desirable
  - Burden on grid / CAD software
  - Discontinous operations present extra challenges
    - Face-edge swapping
    - Point addition / removal
    - Mesh regeneration

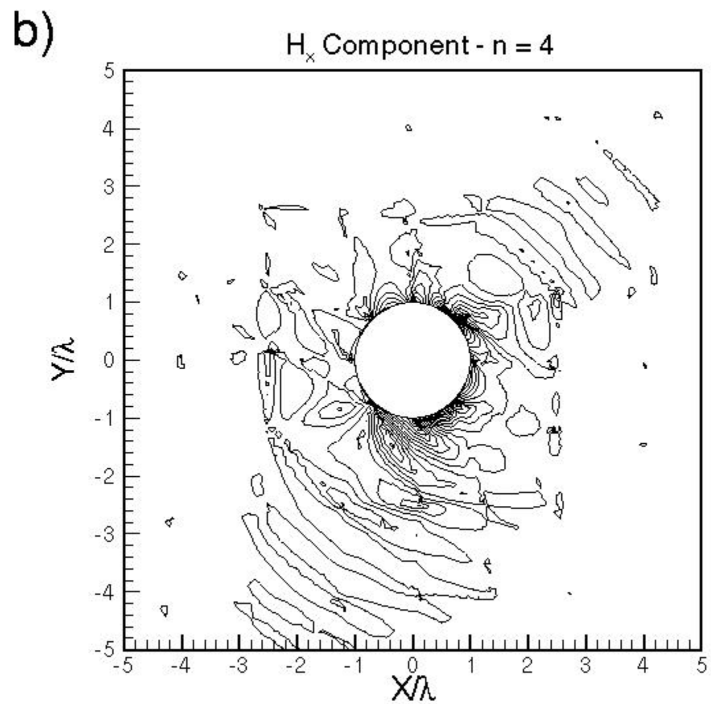
High-Order Accurate Discretizations

- Uniform X2 refinement of 3D mesh:
  - Work increase = factor of 8
  - 2nd order accurate method: accuracy increase = 4
  - 4th order accurate method: accuracy increase = 16
    - For smooth solutions
- Potential for large efficiency gains
  - Spectral element methods
  - Discontinuous Galerkin (DG)
  - Streamwise Upwind Petrov Galerkin (SUPG)

Higher-Order Accurate Discretizations

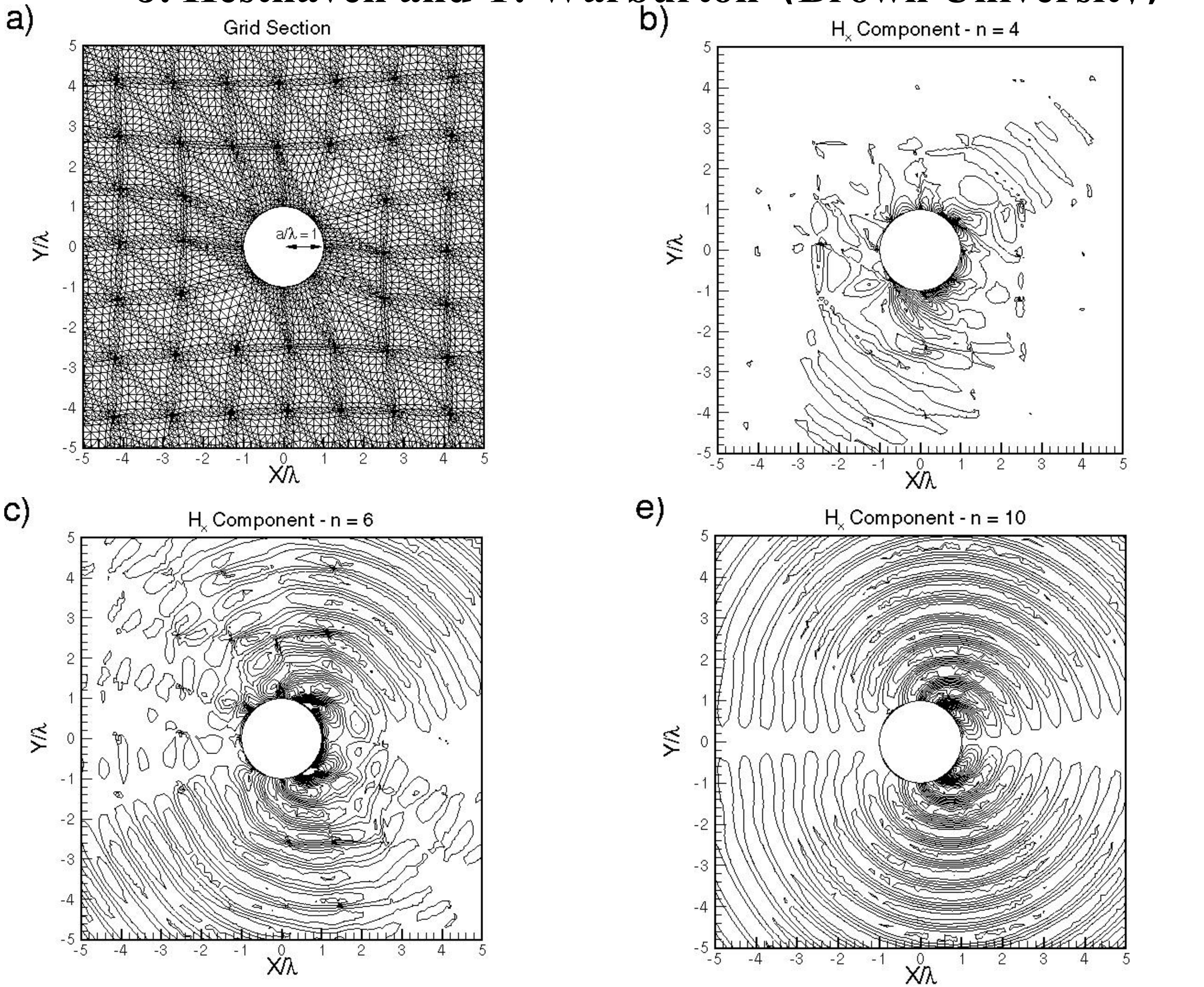
- Transfers burden from grid generation to Discretization





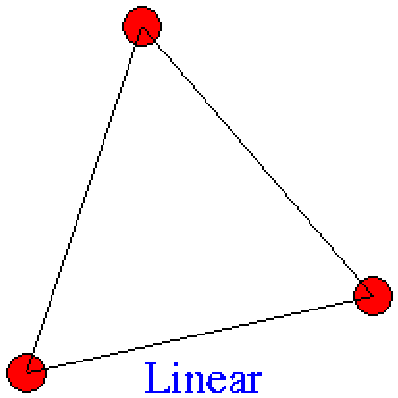
Spectral Element Solution of Maxwell's Equations

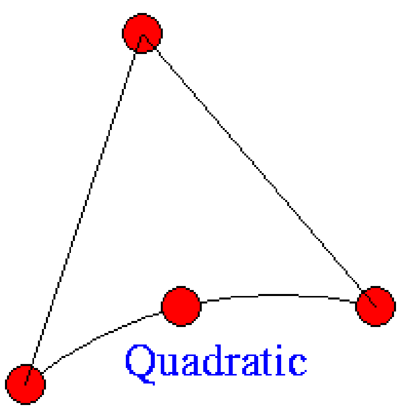
J. Hesthaven and T. Warburton (Brown University)

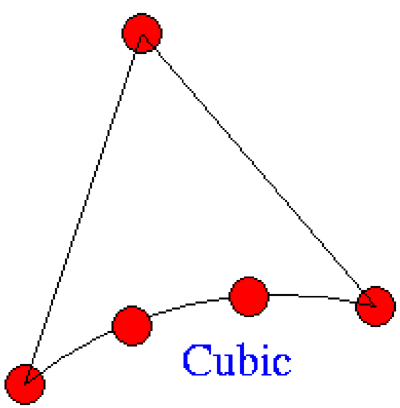


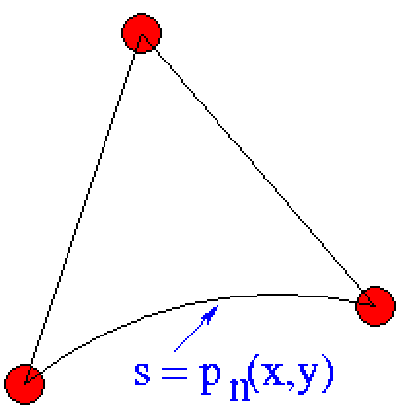
High-Order Discretizations

- Require more complete surface definition
- Curved surface elements
  - Additional element points
  - Surface definition (for high p)









Combined H-P Refinement

- Adaptive meshing (h-ref) yields constant factor improvement
  - After error equidistribution, no further benefit
- Order refinement (p-ref) yields asymptotic improvement
  - Only for smooth functions
  - Ineffective for inadequate h-resolution of feature
  - Cannot treat shocks
- H-P refinement optimal (exponential convergence)
  - Requires accurate CAD surface representation

Conclusions

- Unstructured mesh CFD has come of age
  - Combined advances in grid and solver technology
  - Inviscid flow analysis (isotropic grids) mature
  - Viscous flow analysis competitive
- Complex geometry handling facilitated
- Adaptive meshing potential not fully exploited
- Additional considerations in future
  - Design methodologies
  - New discretizations
  - New solution techniques
  - H-P Refinement